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Modified Vietoris theorems for homotopy.

S. Smale’s Vietoris theorem for homotopy [Proc. Amer. Math. Soc. 8 (1957), 604–610; MR0087106 (19,302f)] imposes local connectivity conditions on the fibers of the given map \( p: X \rightarrow Y \); the present paper offers versions that depend on the manner in which the fibers of \( p \) are embedded in \( X \), rather than on their actual structure. These versions result from a careful study of the homotopy condition on the embedding of the fibers which was considered by T. M. Price [Notices Amer. Math. Soc. 14 (1967), 274, Abstract 67T-197]: A subset \( A \) of a \( T_2 \) space \( X \) is called \( \text{PC}_X^n \) if for each neighborhood \( U \) of \( A \) in \( X \) there is a neighborhood \( V \) of \( A, V \subset U \), such that each map of an \( r \)-sphere into \( V \) has an extension mapping the \((r+1)\)-cell into \( U \), \( 0 \leq r \leq n \).

Applications include a proof of the generalization of Smale’s theorem announced by G. Kozlowski [ibid. 15 (1968), 560, Abstract 68T-406] plus the theorems quoted below, on homotopy excision and on Serre fibrations:

Let \( X \) be paracompact and \( A \subset X \) a closed \( \text{PC}_X^n \) subset. Let \( p: X \rightarrow X/A \) be the projection. If \( X/A \) is dominated by a polytope, then \( p_*: \pi_i(X) \rightarrow \pi_i(X/A) \) is an isomorphism for \( 0 \leq i \leq n \) and is epic for \( i = n + 1 \).

Let \( E \) be compact, \( B \) a polytope and \((E, p, B)\) a Serre fibration. Then every fiber is \( \text{PC}_X^n \) if and only if every fiber is \( n \)-connected.

Reviewed by George McCarty

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