§ 8.2: Integration by Parts (pg.561)

Indefinite: 
\[ \int f(x) g'(x) \, dx = f(x) g(x) - \int f'(x) g(x) \, dx \]
\[ uv = \int uv' - \int v du \]

Definite: 
\[ \int_{a}^{b} f(x) g'(x) \, dx = f(x) g(x) \bigg|_{a}^{b} - \int_{a}^{b} f'(x) g(x) \, dx \]

\text{Limits don't change since } x \text{ is still our variable.}

Tabular Integration:
\[
\begin{array}{ccc}
\text{ex.} & \int x^3 \sin x \, dx & f(x) = x^3 \\
\text{Derivative of } f(x) & g(x) = \sin x & \text{Integrate } \sin x \, dx \rightarrow \cos x \\
x^3 & - & - \\
3x^2 & \rightarrow & \sin x \\
x & \rightarrow & -
\end{array}
\]
Exam 1 Review

Reduction formula: Can handle higher powers of sine and cosine.

§8.3: Integration of Rational Functions by Partial Fractions

\[ \int \frac{f(x)}{g(x)} \, dx \]

Limits won't change since x is still your variable.

Check
1. \( \deg(f(x)) < \deg(g(x)) \). Rewrite if not. \( A(x) + \frac{f(x)}{g(x)} \)
2. Factor \( g(x) \)
3. Cancel terms in top/bottom if possible
4. Setup

Distinct linear factors:
\[ \frac{A}{x-r_1} + \frac{B}{x-r_2} + \ldots \]

Repeated linear factors:
\[ \frac{A}{x-r_1} + \frac{B}{(x-r_1)^2} + \frac{C}{(x-r_1)^3} \]

Nonlinear factors:
\[ \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+x+1} + \frac{Ex^2+Fx+G}{x^3+x^2+x+1} \]

when written with unknown constants,
\[ \deg(\text{num}) = \deg(\text{denom}) - 1. \]
Exam 1 Review

**Exercise:** \( \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \)

\[ 1 = A(x^2+1) + (Bx+C)x \]

**Collect terms of like powers:**

\[ 0x^2 + 0x + 1 = A + Cx + (A+B)x^2 \]

\[ 1 = A \]
\[ 0 = C \]
\[ 0 = A + B \Rightarrow B = -1 \]

**Pick values for \( x \): take derivatives:**

\[ x = 0: \quad 1 = A(1) + C \]
\[ A = 1 \]

\[ D \]
\[ 0 = A(0) + 2Bx + C \]
\[ x = 0: \quad 0 = 0 + 0 + C \Rightarrow C = 0 \]
\[ x = 1: \quad 0 = 2A + 2B+C \]
\[ = 2 + 2B \]
\[ \Rightarrow B = -1 \checkmark \]

**Pick values of \( x \) and solve system of linear equations.**

\[ x = 0: \quad 1 = A \]
\[ x = 1: \quad 1 = 2A + B + C = 2 + B + C \]
\[ B + C = -1 \]
\[ x = -1: \quad 1 = 2A + B - C = 2 + B - C \]
\[ -1 = B - C \]
Exam Review

\[ BtC = 1 \]
\[ + * (\frac{B-C=-1}{\text{or } B = -2}) \]
\[ B = \square - 1 \]
\[ B = 2 \]
\[ - \square + C = -1 \]
\[ C = 0. \]

* Heavy side Method: Only for distinct linear factors.

\[ A_n = \frac{f(r_n)}{(r_n - r_1)(r_n - r_2) \ldots (r_n - r_{n-1})} \]

\[ A_i = \frac{f(r_i)}{(r_i - r_2)(r_i - r_3) \ldots (r_i - r_n)} \]
Exams Review

8.4 \text{ Trigonometric Integrals (pg 581)}

\int \sin^m x \cos^n x \, dx

* If \( m \) is odd: \( \text{let } \sin^m (x) = (1 - \cos^2 x)^{m/2} \), let \( u = \cos x \).

* If \( n \) is odd: \( \text{let } \cos^m (x) = (1 - \sin^2 x)^{m/2} \), let \( u = \sin x \).

* Both even: Sub \( \sin^2 x = \frac{1}{2}(1 - \cos 2x) \) and \( \cos^2 x = \frac{1}{2}(1 + \cos 2x) \), then try again.
Repeat until one power is odd.

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Eliminating \( \sqrt{2} \) Roots

\[
\sqrt{1 + \cos 4x} = \sqrt{2 \cos^2 2x} = \sqrt{2} |\cos 2x|
\]

\( \sec \cosec \): \( \int \sec^m x \, dx \):

- \( m \) even: \( \text{let } \sec^m x = (1 + \tan^2 x)^{m/2} \), let \( u = \tan x \).
- \( m \) odd: \( \text{let } \sec^m x = (1 + \tan^2 x)^{m/2} \),

Do it by parts with \( u = \sec x \) and \( dv = (1 + \tan^2 x)^{m/2} \sec x \, dx \).

\[
\frac{d}{dx} \tan x
\]
Products of Sines & Cosines

\[
\sin(mx) \sin(nx) = \frac{1}{2} (\cos((m-n)x) - \cos((m+n)x))
\]
\[
\sin(mx) \cos(nx) = \frac{1}{2} (\sin((m-n)x) + \sin((m+n)x))
\]
\[
\cos(mx) \cos(nx) = \frac{1}{2} (\cos((m-n)x) + \cos((m+n)x))
\]

SS
SC
CC

Same \rightarrow \text{get cosine}

Diff \rightarrow \text{get sine}

\*

One or more cosines \rightarrow +
No cosine \rightarrow -

§ 8.5: Trig Substitution

\[
x^2 + a^2 \rightarrow \tan \theta = x
\]
\[
x^2 - a^2 \rightarrow \sec \theta = x
\]
\[
a^2 - x^2 \rightarrow \sin \theta = x
\]

Know how to draw triangles!

Change limits if you change \(x \rightarrow \theta\).