Problem Section
Solve each of the following problems. Show all of your work. (i.e. u = 1-5x, etc.)

1. (5 points each) Find the derivative of each of the following functions. You do not need to simplify your answer.
   
   (a) \( f(x) = \frac{\cos x + \sqrt{x}}{x} \)
   
   \[
   f'(x) = \frac{x(\cos x + \sqrt{x})' - (x)'(\cos x + \sqrt{x})}{x^2}
   \]
   
   \[
   = \frac{x(-\sin x + \frac{1}{2}x^{-\frac{1}{2}}) - 1(\cos x + \sqrt{x})}{x^2}
   \]
   
   \[
   = \frac{-x \sin x + \frac{1}{2}x^{-\frac{1}{2}} - \cos x - \sqrt{x}}{x^2}
   \]
   
   \[
   = \frac{-x \sin x - \cos x - \sqrt{x}}{x^2}
   \]

   (b) \( f(x) = \sqrt{x + \sqrt{x}} \)
   
   \[
   f(x) = (x + x^{\frac{1}{2}})^{\frac{1}{2}}
   \]
   
   \[
   f'(x) = \frac{1}{2} (x + x^{\frac{1}{2}})^{-\frac{1}{2}} \left( 1 + \frac{1}{2} x^{-\frac{1}{2}} \right)
   \]

   (c) \( f(x) = \cos^2(\cos x) + \sin^2(\cos x) \)
   
   \[
   f(x) = 1
   \]
   
   Recall \( \cos^2 x + \sin^2 x = 1 \).
   
   \[
   f'(x) = 0.
   \]
   
   You could do the chain rule here:
   
   \[
   f(x) = (\cos(\cos x))^2 + (\sin(\cos x))^2
   \]
   
   \[
   f'(x) = 2 \cos(\cos x) (-\sin(\cos x)) (-\sin x) + 2 \sin(\cos x)(\cos(\cos x)) (-\sin x)
   \]
   
   \[
   = 2 \cos(\cos x) \sin(\cos x)(\sin x) + 2 \cos(\cos x) \sin(\cos x)(\sin x)
   \]
   
   \[
   = 0.
   \]
2. (5 points) Oh no! You are taking the second midterm and you realize that you’ve completely forgotten what the derivative of \( \tan x \) is! Find the derivative of \( f(x) = \tan x \). Just stating what it is will not be sufficient. You have to “discover” it for yourself through some method. You should simplify \( f'(x) \) into a form that we would recognize.

Recall that
\[
\tan x = \frac{\sin x}{\cos x}
\]

\[
\frac{d}{dx} [\sin x] = \cos x
\]

\[
\frac{d}{dx} [\cos x] = -\sin x
\]

\[
f(x) = \tan x = \frac{\sin x}{\cos x}
\]

\[
f'(x) = \frac{\cos x (\sin x)' - \sin x (\cos x)'}{(\cos x)^2}
\]

\[
= \frac{\cos x \cdot \cos x - \sin x (-\sin x)}{(\cos x)^2}
\]

\[
= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2}
\]

\[
= \frac{1}{(\cos x)^2}
\]

\[
= \sec^2 x.
\]

Thus, the derivative of \( \tan x \) is \( \sec^2 x \).