There are 10 points possible. Partial credit may be given where applicable.

Solve each of the following problems. Show all of your work. (i.e. $u = 1-5x$, etc.) If you run out of room, you may use additional sheets of paper. Be aware of problems on the back of the page.

1. (5 points) Find the absolute maximum and minimum values of

$$g(x) = -\sqrt{5-x^2}$$

on the interval $-\sqrt{5} \leq x \leq 0$ and say where they are assumed.

$$g'(x) = -\frac{1}{2} (5-x^2)^{-\frac{1}{2}} (5-x^2)' = -\frac{1}{2} (5-x^2)^{-\frac{1}{2}} (-2x)$$

$$= \frac{x}{\sqrt{5-x^2}}$$

Critical pts: $x=0, \sqrt{5}, -\sqrt{5}$.

Only $x=0$ and $x=-\sqrt{5}$ are in our interval, so we discard $x=\sqrt{5}$.

$$g(0) = -\sqrt{5-0^2} = -\sqrt{5}$$

$$g(-\sqrt{5}) = -\sqrt{5-(-\sqrt{5})^2} = -\sqrt{5-5} = 0.$$ 

Absolute max @ $(-\sqrt{5}, 0)$

Absolute min @ $(0, -\sqrt{5})$
2. (5 points) For what values of \(a\), \(m\) and \(b\) does the function
\[
f(x) = \begin{cases} 
3 & \text{if } x = 0 \\
-x^2 + 3x + a & \text{if } 0 < x < 1 \\
mx + b & \text{if } 1 \leq x \leq 2
\end{cases}
\]
satisfy the hypotheses of the Mean Value Theorem on the interval \([0, 2]\)?

- **Want**
  - \(f(x)\) continuous on \([0, 2]\)
  - \(f(x)\) differentiable on \((0, 2)\)

\[
f(0) = \lim_{x \to 0^+} f(x)
\]

\[
3 = \lim_{x \to 0^+} (-x^2 + 3x + a)
\]

\[
3 = 0 + 0 + a
\]

\[
3 = a
\]

\[
\lim_{x \to 1^{-}} f(x) = \lim_{x \to 1^{-}} (-x^2 + 3x + 3)
\]

\[
m + b = -1 + 3(1) + 3
\]

\[
m + b = 5
\]

- **Differentiable**

\[
f'(x) = \begin{cases} 
-2x + 3 & 0 < x < 1 \\
? & x = 1 \\
m & 1 < x < 2
\end{cases}
\]

\[
\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x)
\]

\[
\lim_{x \to 1^+} (m) = \lim_{x \to 1^-} (-2x + 3)
\]

\[
m = -2(1) + 3 \Rightarrow m - 2 = 1
\]

\[
a = 3
\]

\[
m = 1
\]

\[
b = 4
\]