PHYS-212 Lab Assignment #1

Taylor problem 3.23

In an experiment on conservation of angular momentum, a student needs to find the angular momentum \( L \) of a uniform disc of mass \( M \) and radius \( R \) as it rotates with angular velocity \( \omega \). She makes the following measurements:

\[
M = (1.10 \pm 0.01) \text{kg} \\
R = (0.250 \pm 0.005) \text{m} \\
\omega = (21.5 \pm 0.4) \text{rad/s}
\]

and then calculates \( L \) as \( L = \frac{1}{2} MR^2 \omega \). (The factor \( \frac{1}{2} MR^2 \) is just the moment of inertia of the uniform disc.) What is her answer for \( L \) with its uncertainty? (Consider the three original uncertainties independent and remember that fractional uncertainty in \( R^2 \) is twice that in \( R \).)

**Solution:**

The calculated quantity \( L \) is the product of three factors, \( M \), \( R^2 \), and \( \omega \). The easiest approach then is to focus on the relative or fractional uncertainty in \( L \). We know from differential calculus that

\[
\frac{\delta L}{L} \approx \frac{\delta M}{M} + 2 \frac{\delta R}{R} + \frac{\delta \omega}{\omega}.
\]

These fractional errors are simply given by

\[
\frac{\delta M}{M} = \frac{0.01}{1.10} \approx 1\% \\
\frac{\delta R}{R} = \frac{0.005}{0.250} \approx 2\% \\
\frac{\delta \omega}{\omega} = \frac{0.4}{21.5} \approx 2\%
\]

Since the uncertainties are assumed to be independent, we actually add them in quadrature, or

\[
\frac{\delta L}{L} \approx \sqrt{\left(\frac{\delta M}{M}\right)^2 + 2\left(\frac{\delta R}{R}\right)^2 + \left(\frac{\delta \omega}{\omega}\right)^2}
\]

\[
\frac{\delta L}{L} \approx \sqrt{(1\%)^2 + 2(2\%)^2 + (2\%)^2} \approx 4\%
\]

Hence there is about a 4\% uncertainty in \( L \), mostly due to the uncertainty in \( R \).

Thus \( L = \frac{1}{2} MR^2 \omega = \frac{1}{2}(1.10)(0.250)^2 (21.5) = 0.739 \text{kg} \cdot \text{m}^2/\text{s} \) with an uncertainty of 4\% or 0.03.

To reflect significant figures we write this as

\[
L = (0.74 \pm 0.03) \text{kg} \cdot \text{m}^2/\text{s}
\]