Chapter 3: the Solar Spectrum

Most of the energy used in the world comes from the burning of natural gas, coal, oil, or wood. Ultimately, however, this energy comes to us from the sun. This is true even for hydroelectric power. It is, of course, obviously the case for renewable energy such that generated by photovoltaic arrays or wind power. Thus, to understand the generation of power we must first look at the source of all power, the sun.

At the core of the sun mass is converted to energy via fusion. The sun is so hot that all atoms are fully ionized -- it is a "soup" of nuclei, mainly isotopes of hydrogen and helium. The smaller nuclei combine to form larger nuclei whose mass is slightly less than the sum of the masses of the smaller nuclei. The mass-loss is converted directly to energy ala $E = mc^2$. This is fusion -- it is the basic process which powers stars.

When we look at the sun we see its outer surface. While the interior of the sun is at millions, even billions of degrees, the outer surface is much cooler, close to 6000 K

1. the Stefan-Boltzmann Law

All objects radiate energy in the form of electromagnetic waves. The amount of energy radiated and the exact wavelengths radiated are determined by the body's temperature. This kind of radiation is called "blackbody radiation."[1] The total power radiated by a body of surface area A and absolute temperature T is given by the *Stefan-Boltzmann law*, namely

$$P = \alpha\sigma A T^4, \tag{1}$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant and α is called the *emissivity*. The emissivity is a dimensionless "fudge factor" which ranges from 0 to 1. It accounts for any tendency that the body's surface has to preferentially transmit some wavelengths and not others. A perfect absorber/emitter has $\alpha = 1$, and for simplicity, we will limit our discussion here to such bodies. This assumption will be relaxed later when we discuss the optical properties of windows. The Stefan-Boltzmann Law holds for any object regardless of its temperature. Note that a small increase in temperature leads to a large increase in radiated power.

Example 1:
The sun has a radius $R = 6.95 \times 10^8$ m and a surface temperature of 5800 K. Calculate the total power radiated by the sun's surface, treating it as a perfect radiator.
Solution:
This is a simple application of the Stefan-Boltzmann law with $\alpha = 1$.

$P = \sigma A T^4$ where $A = 4 \pi R^2$, and R is the sun's radius.

$= (5.67 \times 10^{-8}\ \mathrm{Wm^{-2}\ K^{-4}}) \times (4 \pi) \times (6.95 \times 10^8\ \mathrm{m})^2 \times (5800\ \mathrm{K})^4$

$= 3.89 \times 10^{26}$ W.

[1] The origin of the name has to do with the fact that the radiation emitted by an object is intimately related to the radiation it will absorb. A "black" object, in this sense, is an object whose surface will absorb absolutely any radiation which is incident upon it, no matter the wavelength. The quintessential example of this is an object whose surface has been coated with "lamp black," soot from a candle or lamp. While not obvious, it can be shown that the surface of such an object will also emit any radiation which emanates from within, again, no matter the wavelength. Better terminology might be to call these things perfect emitters (and, accordingly perfect absorbers).

2. the Planck Distribution

Hot objects radiate electromagnetic waves throughout the entire electromagnetic spectrum. The amount of energy radiated in given range of the spectrum depends on the temperature. The hotter the object, the more the radiated energy shifts to shorter wavelengths. The cooler the object, more of its energy is radiated at longer wavelengths. The sun's radiation is mostly in the visible spectrum, peaking near the wavelength of yellow light. The earth is much cooler. Its radiation is mostly in the infrared spectrum.

The power spectral density of blackbody radiation is quantified by S_λ given by

$$S_\lambda = \frac{2\pi c^2 h}{\lambda^5} \frac{\alpha(\lambda)}{e^{hc/\lambda kT} - 1}, \tag{2}$$

where h = 6.63 x 10^{-34} Js is Planck's constant, and k = 1.38 x 10^{-23} J/K is Boltzmann's constant. Here I have introduced the wavelength-dependent emissivity $\alpha(\lambda)$. For a perfect radiator/absorber, α=1 for all wavelengths. For a very narrow range of wavelength $\Delta\lambda$, the energy flux (i. e., power per unit area) radiated in that range of wavelengths is $S_\lambda \Delta\lambda$. The Planck spectrum for a perfect radiator is plotted below (versus wavelength) for a temperature, T_1 = 6000 K and for one just slightly lower, T_2 = 5000 K.

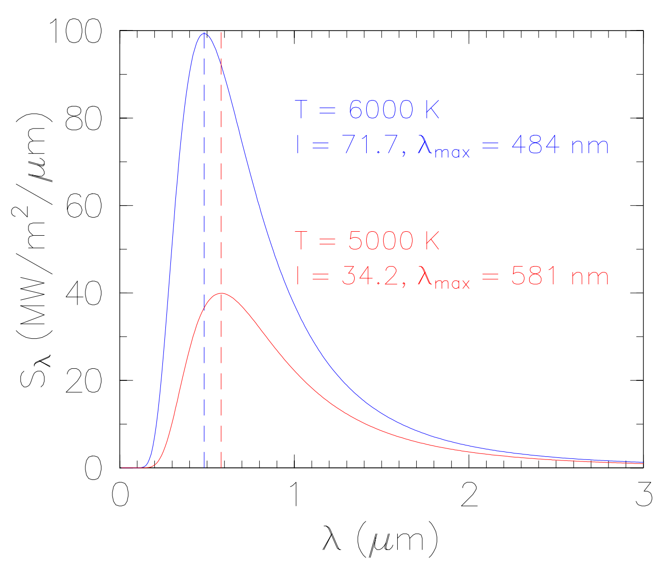

Figure 1. Planck spectrum plotted for two temperatures, 6000 K (blue) and 5000 K (red). The two dashed lines locate the peaks for the two curves. The total fluxes, I, are given in units of 10^6 W/m^2.

Note that a small increase in temperature has two important effects: 1) it makes the curve get much taller (and hence the area under the curve much larger), and 2) it shifts the peak to slightly shorter wavelength. The total flux (accounting for all wavelengths) is given by the Stefan-Boltzmann law and represents the total area bound between the horizontal axes and the curve.

Example 2:

Calculate the total radiation intensity from a perfect radiator at a temperature T = 5000 K.

Solution:

This is a simple application of the Stefan-Boltzmann law. Recall that intensity is just power per unit area.

$$I = P/A = \sigma T^4$$
$$= (5.67 \times 10^{-8}\ Wm^{-2}K^{-4}) \times (5000\ K)^4$$
$$= 3.54 \times 10^7\ W/m^2.$$

Note that this agrees with the total area under the red curve in Figure 2 above.

It is important to understand that every body emits radiation described by the Planck distribution simply due to its temperature. The earth is roughly at 300 K and emits blackbody radiation with much lower intensity. About two decades ago, Wilson and Pensius, who then worked for AT&T Bell Laboratories, discovered that empty space seems to be full of radiation whose spectra is given by Planck's formula for a temperature T = 3 K. This radiation has subsequently been shown the remnant left over from the creation of the universe, the so-called "big bang." A few years ago much more sensitive measurements were performed by the Hubble space telescope confirming that this 3-K radiation is uniform throughout all space.

Before leaving this topic I must confess that I haven't been quite honest. The idealized emitter, the "blackbody," has a spectrum as I have described. Real objects have surfaces which alter the emitted distribution. That is, the surface may not transmit radiation from the interior the same for all wavelengths. For instance, window glass will transmit the visible spectrum without attenuation, but will not transmit infrared wavelengths nearly so well. Thus, both the Stefan-Boltzmann law and the Planck distribution ought to include a "fudge factor" called the emissivity, which makes the appropriate correction. This will be discussed later.

3. Wein's Displacement Law

The above distribution is, quite frankly, pretty hard to deal with. At any temperature the distribution will look qualitatively like Figure 1. What will change will be the height of the curve and the location of the peak. The area under the curve is related to the height. The area is given by the Stefan-Boltzmann formula and increases with the 4th power of T. This means that the curve gets higher, a lot higher, with increasing temperature.

The other thing that changes with temperature is the position of the peak, that is, the wavelength at which the flux density is maximum. For the solar spectrum in Figure 1 this peak wavelength occurs at 481 nm. This peak wavelength, λ_{max} inversely with temperature, that is

$$\lambda_{max} T = \gamma \quad , \qquad (3)$$

where $\gamma \approx 2898$ µm·K.

Example 3:
The earth is at a temperature of 300 K. It, too, radiates blackbody radiation. Calculate the maximum wavelength for the earth's blackbody radiation.

Solution:

$\lambda_{max} = \gamma/T$
$= (2.90 \text{ x } 10^{-3} \text{ m K}) / (300 \text{ K})$
$= 9.7$ mm.

This is in the infrared. This is crucial for understanding the green-house effect.

4. The Solar Spectrum

The radiation from the sun may be modeled by that of a black body at a temperature of about 6000 K. Measurements of the solar radiation are made at the earth (not at the surface of the sun) so are lower intensity than that given by the Planck spectrum by the ratio $(R_S/r_{es})^2$, where R_S is the sun's radius and r_{se} is the mean distance between the earth and sun. Moreover, the sun's radiation has to pass through the earth's atmosphere before reaching the surface which slightly reduces its intensity. The spectrum is plotted in the figure below reduced by the appropriate geometric factor. The blue curve is the theoretical spectrum and the green curve is the actual measured spectrum. Note that absorption in the atmosphere removes certain wavelengths nearly entirely.

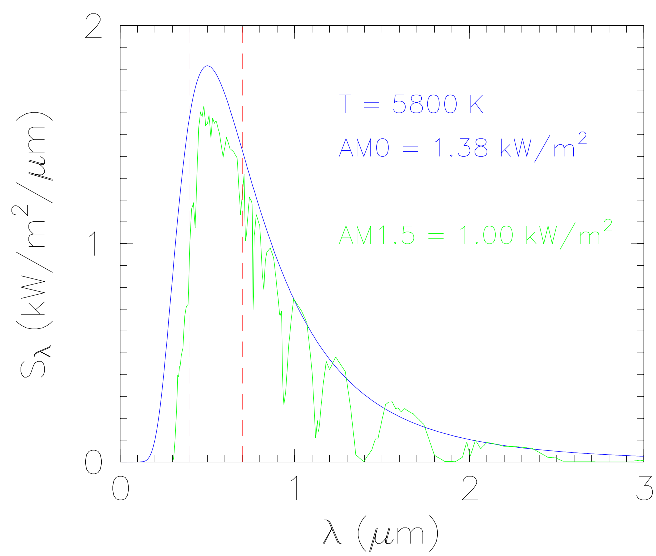

Figure 2. The blue curve (smooth) is calculated from the above formula for a temperature T = 6000 K. The green curve (jagged) is the measured solar spectrum. Both curves are adjusted for distance from the sun. The dashed vertical lines indicate the visible spectrum, 400 nm - 700 nm.

The area bounded between the curve and the horizontal axis is the total power per unit area at all wavelengths. This is 1350 W/m^2. The area bounded by the curve on top, the horizontal axis on the bottom, and the two dashed lines, is the amount of power per unit area in the visible range, 520 W/m^2. The amount of radiation in the ultraviolet (below 400 nm) and infrared (above 700 nm) ranges are 192 and 640 W/m^2 respectively.

5. Detailed Balance -- the Energy Budget

The earth is constantly receiving radiation from the sun. If that were the end of the story then the earth would be continually heating up -- i. e., global warming. In fact, this does not happen because the earth is also giving off radiation -- blackbody radiation of its own, but characteristic of its lower temperature, T_E. In a given amount of time, if the earth receives more energy than it gives up -- then its temperature will rise. If, on the other hand, it gives up more than than it receives, its temperature will decrease. Roughly speaking we are in a very delicate balance, we radiate a total amount of power that is just equal to the amount we receive from the sun. Looking at this delicate balance will help us understand why we are not too cold, or too hot. The process is illustrated below.

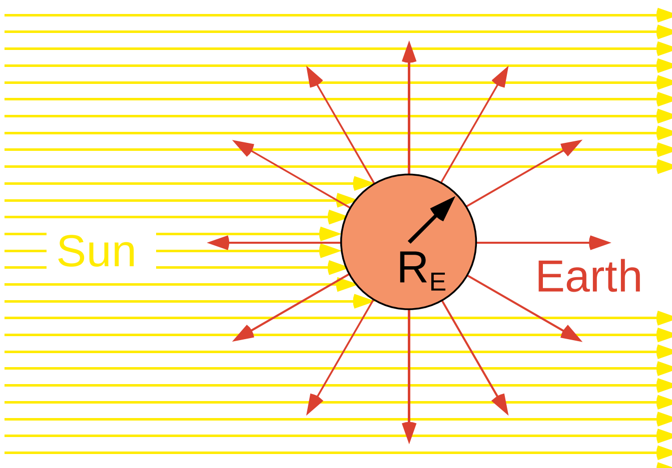

Figure 3. The Sun's radiation is absorbed by only 1/2 of the earth, with an effective area of that of a circle of radius R_E. The Earth gives off IR radiation in all directions.

The earth is a sphere of radius R_E. The entire spherical surface (roughly speaking) is at a temperature T_E. Therefore the entire earth's surface radiates energy away at a rate given by

$$P_{loss} = \sigma A_E T_E^4 = 4\pi\sigma R_E^2 T_E^4 \,. \tag{4}$$

Now let's consider the rate at which energy is received by the sun. If the sun has a radius R_S, a surface temperature T_S, and is a distance r from the earth, then the intensity of the sun's radiation at the earth's surface is

$$I_{sun} = \sigma T_S^4 \left(\frac{r}{R_S}\right)^2 . \tag{5}$$

Now, the area of the earth which receives this radiation is just the area of a circle of radius R_e. Thus, the rate of energy gain by the earth is I_{sun} times this area. For balance, $P_{loss} = P_{gain}$, that is

$$\sigma T_E^4 4\pi R_E^2 = \sigma T_S^4 \left(\frac{R_S}{r}\right)^2 \pi R_E^2 \,. \tag{6}$$

This equation may be solved to find the earth's temperature, namely

$$T_E = T_S \sqrt{\frac{R_S}{2r}} \, . \tag{7}$$

Plugging in T_S = 5800 K, r = 1.5 x 10^{11} m, and R_S = 6.95 x 10^8 m we find T_E = 279 K, just above the freezing point of water. Of course this is an oversimplification, but it does show roughly why T = 300 K in the world we live in. The slightest change in distance from the sun, or atmospheric conditions, etc. would change this delicate balance and shift our temperature accordingly.

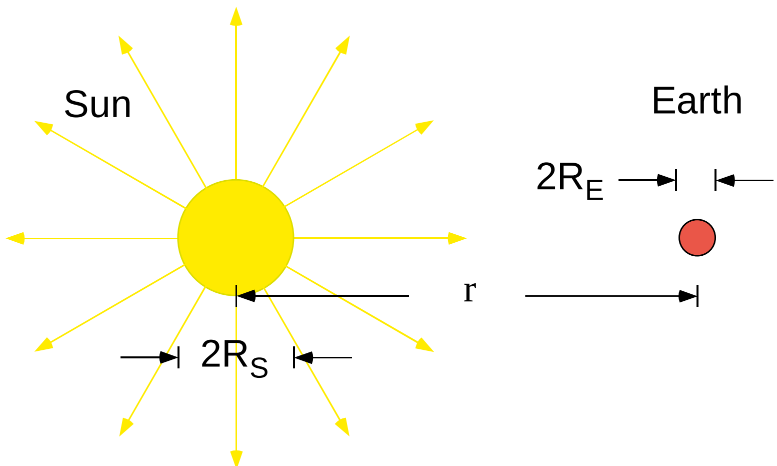

Figure 4. The earth has a radius R_E, the sun a radius R_S, and their separation distance is r.

6. Emissivity

So far we have considered only perfect emitters/absorbers, that is one that absorb any incident radiation incident upon it and emits any radiation that is within it. An object that is black in color with a dull (rather than shiny) finish has this appearance because it is absorbing all incident radiation. Thus, a "black body" approximates an ideal absorber/emitter. A metal with a rough surface that has been coated with "lamp black" (soot from a candle flame) or flat-black paint will absorb nearly 100% of the visible radiation incident upon it.

In contrast, a shiny, mirror-like metallic surface will not absorb much radiation at all. Instead, it will reflect most of the radiation. Since the absorbing and emitting properties of a surface are closely related, such a body will emit only a small fraction, α, of its radiation spectrum emitted by a "perfect radiator" having the same temperature.

Furthermore, real objects have surfaces that won't treat all wavelengths the same. For instance, conventional window glass appears transparent to visible light, but does not transmit infrared or ultraviolet radiation so effectively. Quartz, on the other hand, transmits both visible and UV radiation. This preference for a surface to transmit/reflect some wavelengths better than others is accounted for by its wavelength-dependent *emissivity*. A perfect radiator/absorber has an emissivity α = 1 at all wavelengths. Real bodies have emissivities that may be less than unity, and also, that may vary with wavelength. For instance, a rough, flat surface (i. e., one that appears dull rather than shiny) painted with flat, black paint will have an emissivity very close to unity. On the other hand, a polished aluminum surface that appears mirror-like may have an emissivity as low as 0.01 (i. e., 1%).

7. the Greenhouse Effect

The greenhouse effect refers to the warming that occurs when sunlight passes through the transparent wall of a "greenhouse" and warms the air inside. This effect is also observed in cars, homes, and in the earth at large, where the earth's atmosphere takes on the role of the "window." What happens is this. The sun emits a radiation spectrum which is largely concentrated i the visible region. This radiation passes freely through window glass and is mostly absorbed by any objects within, resulting in their heating. These objects are at a temperature close to 300 K. Like all objects, they too emit a blackbody spectrum, but one which peaks in the infrared owing to their significantly lower temperatures. This radiation would travel out freely except that the window glass does not transmit IR very well, so most of it is reflected back into the interior of the room, further heating up its contents. As the contents heat up their radiation spectra shift to slightly shorter wavelengths and increases in magnitude. As a result, more of this radiation gets through the window. Eventually, the objects reach a temperature T where the rate at which energy is emitted exactly balances the rate at which it is received, i.e., steady state. This occurs, however, when the bodies are at higher T than would be the case if the glass were not reflecting back much of their infrared radiation. The net result is that the interior of the car, home, or greenhouse is much warmer because of this greenhouse effect. The effect is illustrated in the figure below.

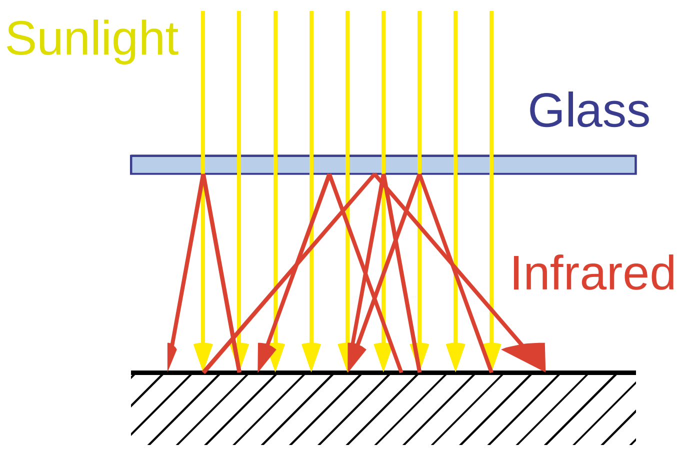

Figure 5. The greenhouse effect results in warmer interior temperatures because the glass lets visible radiation from the sun enter freely, but blocks the infrared radiation from leaving.

This "greenhouse" effect is very important for passive solar heating. During the daylight hours windows which face the sun can provide *solar gain* -- that is, allow more energy to enter via radiation than leave via conduction. On the other hand, this solar gain is not desirable during the summer months. In recent years clever designs have resulted window with emissivities greatly improved for this purpose. Windows will be the subject of a later chapter.

The greenhouse effect also is at work in determining the earth's temperature. Earlier we used detailed balance to calculate the earth's temperature. We found that it was somewhat colder than 300 K. In fact, the earth is surrounded by an atmosphere which, like window glass, tends to transmit visible light and absorb infrared. The optical properties of our atmosphere are far more complicated than simple glass but the principle is the same. The earth's temperature increases substantially over what it would be without the atmosphere.

Moreover, changes in the atmosphere can lead to important changes in the earth's temperature. This is the principle fact behind the global warming debate. This topic will be taken up in a later chapter.

8. the Incandescent Light bulb

The standard light bulb, or incandescent light bulb, uses electric current to heat a wire to a very high temperature, approximately 3000 K. The wire, typically made from tungsten since most other metals would melt at this temperature, gets so hot that it glows brightly. If the wire were exposed to air, which contains oxygen, it would rapidly oxidize and burn up. Instead the wire is contained in a closed glass "bulb" from which the air has been evacuated. The vacuum is not perfect and eventually, after many hours of use, the filament will oxidize.

The hot filament gives off radiation described by the Planck spectrum, shown below. The spectrum for a 100 W bulb is given below (in units of power per unit wavelength). The total area under the curve is 100 W which is equal to the electrical power. Most of the power (92%) is radiated in the infrared (red). Only 8 % is in the visible spectrum (green). Fortunately the glass container is very poor at transmitting infrared radiation, so that some of the infrared radiation generated does not escape.

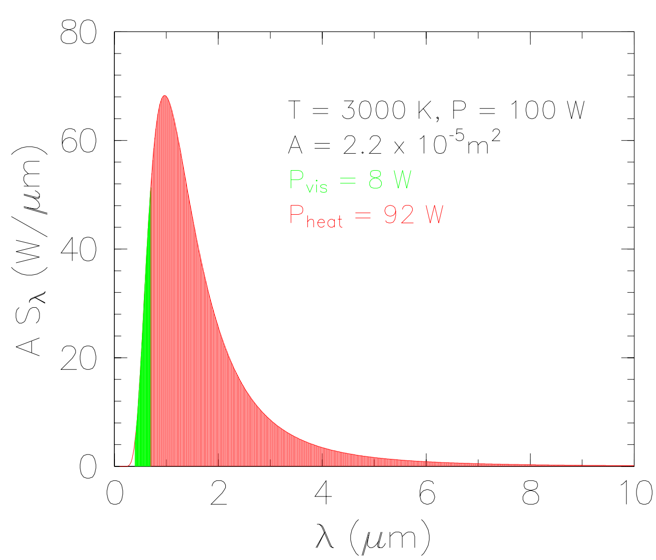

Figure 6. Planck spectrum for the tungsten filament of an incandescent light bulb. Only a small fraction of the power is radiated in the visible spectrum.

Incandescent light bulbs have improved somewhat since the time of Edison. "Energy saver" light bulbs have special coatings on the glass which reduce, even further, the amount of transmitted light in the infrared. Since the bulb radiates less total power, the filament heats up even further, raising its temperature. Or, compared with a bulb that does not have these coatings, it takes less electrical power to heat it up to the same temperature. So, energy-saver bulbs produce the same amount of light as a regular light bulb but with less electrical power.

Another way of improving the efficiency of the incandescent light bulb is to have an even hotter filament. A hotter filament will shift the Planck distribution to shorter wavelength, increasing the fraction of the radiation in the visible range. Problem is, running the filament hotter increases the rate that it oxidizes.[2] This problem is eliminated by filling the bulb with a halogen gas. The tungsten-halogen bulb runs hotter and, accordingly, produces more light than heat relative to the standard light bulb. The hotter bulb introduces new worries of fire hazard

9. Building Radiative Heat Transfer

One of the most useful ways to supply heat to a building is through a radiator – some device that is very hot that will give off heat to the space. This could be electric heater coils, a steam radiator, hot-water fin-tubes, or hot water coils embedded into a concrete floor. All three of the heat-transfer mechanisms are important – conduction, convection, and radiation – but the hotter the “radiator” (relative to the space temperature) the more important is radiation. This is because the rate of heat loss from a hot object varies as T^4.

The efficiency of various radiative heat system depends, then, very strongly on the temperature. In the Lewis Center, for instance, there are several such systems. The atrium is heated by hot water that circulates through plastic tubes embedded in the concrete floor. The living machine is heated, in part, via a perimeter, copper fin-tube system that follows the base of the perimeter. The new Science Center has fin-tube coils embedded at the base of many of its south-facing windows.

Example 4:

Compare the rates at which heat will be emitted by a radiative heat system operated at two possible temperatures, 180ºF and 120ºF.

Solution:

Recall that the rate of heat loss goes as the fourth power of the temperature, expressed in Kelvin. So, we need to convert these two temperatures to Kelvin, then find the ratio of their 4th powers. The conversion is straight-forward

$T_1 = (180ºF - 32ºF)(5K/9ºF) + 273K$
$= 355K.$

$T_2 = (120ºF - 32ºF)(5K/9ºF) + 273K$
$= 322K.$

$(322/355)^4 = 0.68.$

That is, the radiator, if operated at the lower temperature, will supply heat into the space at a rate that is only 68% as much as if it were operated at the higher temperature. This is important when considering.

These kind of calculations are important for the design and performance of a solar hot-water heating system. On a cloudy day the water temperature from solar panels will be considerably lower than for sunny days. This will greatly decrease the efficiency at which the system delivers heat into the living space.

[2] Even though the lightbulb is evacuated it is never possible to remove all of the air. The little oxygen remaining is eventually responsible for the filament burning up.