<u>Chapter 5</u>: the Thermal Envelope

About 40% of the energy used in buildings is associated with heating, ventilation, and air-conditioning. In these northern latitudes any home owner knows that the winter heating bill can be the largest utility bill for the house. Thus, reducing heat losses from a building, and increasing the heat gain through appropriate use of windows, can have a major impact on the amount of energy consumed by a building.

Earlier we discussed how to calculate the heat flow through a slab whose dimensions, thermal conductivity, and temperature difference are known. Here we return to this problem but in a simpler manner, making use of tables listing "R-values" for standard building materials.

1. Thermal Resistances

Fourier's heat law is, in many ways, quite analogous to *Ohm's law*. Recall that the current (I) which flows through an electrical resistor (R) is proportional to the voltage difference (ΔV) across the resistor, namely

$$I = \frac{\Delta V}{R}.$$

For Ohm's law, the quantity that "flows" is electric charge and its rate-of-flow is the current, I. The "force" which drives the flow is a difference in voltage, ΔV; I is proportional to ΔV. The proportionality constant is the resistance R.

In the case of Fourier's heat law the quantity that "flows" is energy (that is, heat) and its rate-of-flow is P. The "force" which drives the heat flow is a temperature difference, ΔT. Similarly, we may define a *thermal resistance*,

$$R_{thermal} \equiv \frac{1}{A}\frac{d}{\kappa},$$

so that Fourier's heat law may be written as

$$P = \frac{\Delta T}{R_{thermal}}.$$

Equations (1) and (3) are analogous. The table below summarizes the corresponding quantities for electrical and heat flow.

Electric	Heat
I	P
ΔV	ΔT
R	$R_{thermal}$

The thermal resistance of a slab having thickness d, thermal conductivity κ, and area A is given by Eq. (2) above. Once we calculate the thermal resistance of the slab, then we can easily calculate the rate of heat that will flow through that slab for any given temperature condition.

It is convenient to think in terms of thermal resistances once we begin combining materials. A wall is typically made by combining layers of different materials. For instance, a home might have an outside wall that consists of four layers: 1/2 in. drywall, 2" x 4" wood studs filled with

fiberglass insulation, 1/2 in. plywood, and aluminum siding. The cross-section of such a wall is diagrammed below.
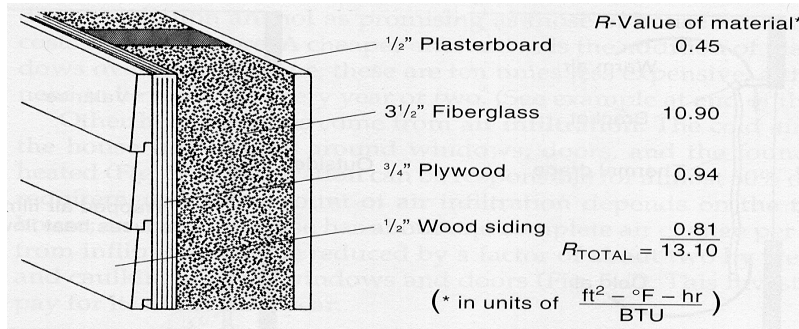

<u>Figure 1</u>.   Cross-sectional view of a typical exterior wall consisting of four different layers.[1] The skeleton of the wall is provided by 2 x 4 studs; the top of one is shown.

For heat to flow from one side of the wall to the other it must flow through each layer in succession -- we say that *the thermal resistances are in series*. The composite wall may be represented by an equivalent thermal resistance that is the sum of the thermal resistances for the individual layers. This is just like electrical resistances in series.

2.   R-Values and Multiple Layers

In order to calculate thermal resistances we must know the wall area -- and since buildings come in all shapes and sizes, there is no way ahead of time to know what A is. Building materials, however, come in standard thicknesses. We define the R-value of a given material to be its thickness divided by its thermal conductivity,[2] namely

$$R \equiv \frac{d}{\kappa}.$$

Thus, we say that a basement wall made from 8-in thick cement blocks has a certain R-value. Similarly, 1/2-inch-thick wall board has another R value. Using the R-value of a particular construction material, the heat flow through a slab of that material having area A is given by

$$P = \frac{A}{R} \Delta T \ .$$

The R-values for a number of common building materials are summarized in the Table I below.[3]

In the United States wall areas are usually specified in square feet, temperature differences in degrees Fahrenheit, and heat flow rates (P) are given in Btu per hour. Accordingly, the R-

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[1]   From Roger A. Hinrichs, *Energy: Its Use and the Environment*, 2nd ed. (Saunders, New York, 1996), p. 127.

[2]   This approach to R-values only allows for thermal conduction. In fact, convection may be even more important for determining heat loss. Tabulated R-values are actually measured and account for heat loss from all mechanisms, not just conduction.

[3]   From Roger A. Hinrichs, *Energy: Its Use and the Environment*, 2nd ed. (Saunders, New York, 1996), p. 125.

value is usually given in these units. The above equation for heat flow is similar to Ohm's law for electrical current.

One of the nice characteristics of *R-values* is the way they add. If a wall is composed of two layers, one layer having an R-value R_1 and the other having an R-value R_2 then the combined layers have an *effective R-value* given by

$$R_{eff} = R_1 + R_2 .$$

This is analogous to series resistance for electrical circuits. (Like the current in a series circuit, the same heat must flow through each of the sequential layers of the wall.) Successive thermal resistances add just like series resistances add in an electrical circuit.

Material	Thickness (inches)	R-Value (ft^2-hr-°F/Btu)
hardwood	1	0.91
softwood	1	1.25
plywood	1/2	0.62
concrete block	8	1.04
common brick	1	0.20
sheetrock (gypsum board)	1/2	0.45
fiberglass insulation	3-1/2	10.9
fiberglass insulation	6	19.0
expanded polystyrene board	1	4.0
expanded polyurethane board	1	6.3
cellulose insulation	1	3.7
"thermax" or "high-R" sheathing	1	8.0
flat glass	1/8	0.88
insulating glass	1/4 of air	1.54
insulating glass	1/4 of air	1.72
wood subfloor	25/32	0.98
hardwood flooring	3/4	0.68
nylon carpet	1	2.0
tile		0.05
asphalt roof shingle		0.44
asbestos shingle		0.21
steel	1	0.0032
copper	1	0.0004
wood siding (lapped)	1/2	0.81

Table I. R-values for a variety of common construction materials.

Example 1:

The walls of a "super insulated home" are to have an R-value of 40. What thickness of fiberglass insulation will it take to achieve this? What thickness of polystyrene board will achieve the same?

Solution:

In Table I above we see that 6 inches of fiberglass has an R-value of 19. Thus, it will take roughly 12 inches (12.6 inches, to be exact) to achieve the desired value.

In the case of polystyrene board, a 1-in-thick board has an R-value of 4, so it will take 10 of these to achieve R-40.

3.   Parallel Heat Flow and U-Values

We now know how to calculate the heat flow through a multi-layered wall. What do we do, however, with a wall for which different areas of the wall are made from different materials? For instance, consider an 8 ft. x 16 ft. wall in which there is a 4 ft. x 3 ft. window. This situation is analogous to an electrical circuit consisting of <u>parallel resistors</u>. We simply calculate the rate of heat flow through the window, and the rate of heat flow through the rest of the wall, then add the two to get the total rate of heat flow.
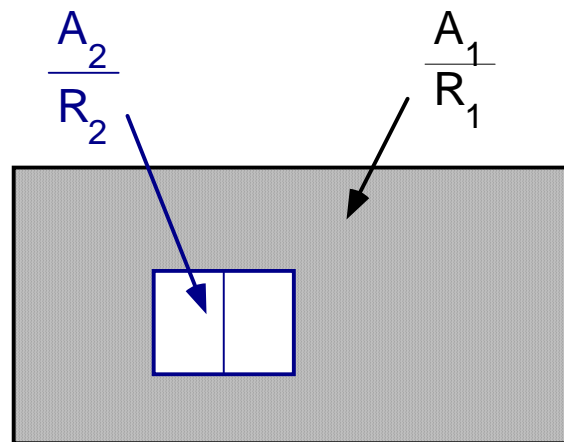

Figure 2. Wall with total area $A_1 + A_2$ containing a window of area A_2. The non-window portion of the wall has area A_1. The R-value for the window is R_2 and the R-value for the non-window portion of the wall is R_1.

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Example 2:
Calculate the total heat flow through an 8 ft. x 16 ft. wall which contains a 4 ft. x 3 ft, single pane glass window on a typical winter day. The non-window portion of the wall is constructed from wood siding on the outside nailed to 3/4 in. plywood, followed by 3-1/2 in. wood stud walls filled with fiberglass insulation, with 1/2-in. drywall on the inside. You may assume that the indoor and outdoor temperatures are 70°F and 30°F respectively.

Solution:
The area of the non-window portion of the wall is
A_1 = (8 ft. x 16 ft) - (4 ft. x 3 ft.)
     = 116 ft^2
From Table I we find the R-values for the wood siding, 1/2-in plywood, insulation, and drywall to be 0.81, 0.62, 10.9, and 0.45. The effective R-value for the four layers is
R_1 = (0.81 + 0.93 + 10.9 + 0.45)
     = 13.1 ft^2-hr-°F/Btu
The heat flow through the non-window portion of the wall is then
P_1 = ΔT (A_1/R_1) = (40°F) (116 ft^2) / (13.1 ft^2-hr-°F/Btu)
     = 354 Btu/hr
The heat flow through the window is calculated similarly. From Table I we find that the single pane glass window has an R-value of 0.88. Thus
P_2 = ΔT (A_1/R_1) = (40°F) (12 ft^2) / (0.88 ft^2-hr-°F/Btu)
     = 545 Btu/hr
The total heat loss rate then is the sum of these two,
P   = 899 Btu/hr.
Thus, though much less in area, the window is responsible for more than half of the heat loss through this wall. This illustrates the basic problem with windows. It's nice to get their light, but at a terrific cost!

When one examines the heat loss from doors and windows we find that we are adding these up. It is, therefore, convenient to think in terms of the U-value, which is simply the inverse of the R-value,

$$U \equiv \frac{1}{R} = \frac{\kappa}{d}.$$

The heat loss rate P through a window having an area A and U-value U is then given by
$$P = UA\Delta T.$$

The total heat loss rate from several windows having areas A_1, A_2, A_3 ... and U-values U_1, U_2, U_3 ..., then is given by
$$P = (U_1 A_1 + U_2 A_2 + U_3 A_3 + ...)\Delta T.$$

The quantity in parenthesis is called the *total UA product*. Below are tables of U-values for a variety of doors and windows commonly used in construction. The usual units for U-values are Btu/(ft^2-hr-°F).

Solid Wood	No Storm	Wood Storm	Metal Storm

1 in.	0.62	0.30	0.38
1-¼ in.	0.53	0.28	0.34
1-½ in.	0.47	0.27	0.33
2 in.	0.42	0.24	0.29

Metal Door	Mineral Fiber Core	Solid Urethane Foam Core
1-3/4"	0.57	0.08

Table II. U-values for various door combinations. The units are Btu/(ft^2-hr-°F).[4]

Type	Operable *U-Value*	Fixed *U-Value*	Door *U-Value*	Skylight *U-Value*
Metal without thermal break, single glazing	*1.30*	1.18	*1.27*	1.92
Metal without thermal break, double glazing	0.87	0.69	0.80	*1.30*
Metal with thermal break, single glazing	1.08	1.11	1.10	1.92
Metal with thermal break, double glazing	0.67	0.63	0.66	1.14
Metal-clad wood, single glazing	*0.98*	1.05	0.99	*1.50*
Metal-clad wood, double glazing	*0.60*	*0.58*	*0.57*	0.88
Wood/vinyl, single glazing	0.94	1.04	*0.98*	1.47
Wood/vinyl, double glazing	*0.56*	0.57	*0.56*	0.85
Glass block assembly		*0.60*		

Table III. U-values for various windows (called glazings). The units are Btu/(ft^2-hr-°F).[5]

4.   Total Heat Loads

We now have the tools to calculate the total heat loss given the outside temperature. This requires that we put together a number of the ideas previously presented.

4.1   Total UA Product for Entire Building

Now that we know how to calculate the rate of heat loss through a given wall, we simply apply these principles to each wall, ceiling, and floor of a home to find the total heat load. Of course, to do this we need to know the inside and outside temperatures. It is reasonable to choose an inside temperature close to 70°F, but the outside temperature will vary from day to day, and even throughout a given day. Also, the basement wall and floor are in contact with the earth whose temperature does not change appreciably from 55°F while the temperature outside the walls and roof will vary.

It will be very inconvenient to have to perform a new heat calculation for each day. We need a method for performing most of the calculation without knowing the outside temperature, then finish the calculated heat loss for a given temperature.

This brings us back to the idea of parallel and series resistors. For all of the walls that have the same temperature drop across them (i. e., outside walls and roof), we can write the total heat load in terms of the total UA product, namely

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[4]   Ken Elklund and David Baylon, Design Tools for Energy Efficient Homes, 3rd edition, p. 105
[5]   Model Energy Code, 1993 Edition, Ohio Version. Thermal Envelope Compliance Guide, p.7.

$$P_{tot} = \frac{A_1}{R_1}\Delta T + \frac{A_2}{R_2}\Delta T + \frac{A_3}{R_3}\Delta T + ... = \left(\frac{A_1}{R_1} + \frac{A_2}{R_2} + \frac{A_3}{R_3} + ...\right)\Delta T$$

$$P_{tot} = \left(U_1 A_1 + U_2 A_2 + U_3 A_3 + ...\right)\Delta T \equiv (UA)_{total}\Delta T .$$

While all the above-ground walls are exposed to the same outside temperature, below ground walls have to be handled separately. We can design a second, effective R-value which covers the entire basement portion of the house (the underground part) and, even without knowing the outside temperature, calculate the heat loss rate for the basement.

4.2 Design Temperature

Once we have calculated the heat loss rate from the basement and the effective R-value for the rest of the building we are in a position to calculate the heat loss rate for a given outside temperature. Now what?

Well, for one thing, we will need to use this number to figure out how much heat we must supply on the coldest day of the year. When designing a building this must be done to determine the size of the required furnace, heat pump, or other system for supplying heat. The coldest temperature at which a building is designed to operate with complete comfort is called the heating design temperature. A typical figure for our region of the country is 5 °F. If we were designing a building for southern Florida we would use a much higher temperature.

Similarly, we may need to provide the building with air-conditioning in the hot seasons. The hottest temperature at which the building is designed to operate with complete comfort is called the cooling design temperature. A typical figure for our area is 85 °F.

The maximum heat load, P_{max}, is the rate of heat loss when the outside is at the heating design temperature. In designing a building we must size the furnace (heat pump, or other source of heat) to be able to deliver this maximum power. The peak heat load is given by

$$P_{max} = (UA)_{total}\left(65° F - T_{design}\right) .$$

4.3 Heating Degree Days and Annual Heat Loss

To calculate the actual amount of energy required to heat a building throughout the year we must know the temperature at all times. Recall that $\Delta E \approx P\Delta T$. Suppose that we have a graph of the local outside temperature (T) versus time for the entire year. The net energy loss from the "above-ground" portion of the house to the outside will be

$$\Delta E = \int_{year} P(t)dt = \frac{A_{tot}}{U_{tot}}\int_{year}\Delta T(t)dt = (UA)_{total}\int_{year}\Delta T(t)dt .$$

In words, the total energy is equal to the total UA product for building times the "area" under a graph of ΔT versus time. It is easier to talk about this in terms of average temperature.

But this is not quite what we are after. In the summer, heat flows from the outside into the house. This heat gives a negative contribution to the above expression for "net energy loss" as if it were a "gain." In fact, this kind of heat flow in the summer will cause us to expend energy as well in the form of air-conditioning. Even if we don't use air-conditioning, the contribution in the summer will not lower our winter heating costs because we have no way to store the energy that long.

A house does have some thermal mass (i. e., heat capacity) so that we can store heat energy for a time which does help offset the effects of short term outside temperature changes, say, on the scale of a day or less. To find the amount of energy required to heat the building over a period of one year we adopt the following point of view. First, we calculate the mean temperature, $\overline{T}$, for a given day, defined to be the average of the high and low temperature for that day. (This is convenient because this is readily available in the daily newspaper and is kept by meteorologists.) That is,

$$\overline{T} \equiv \frac{1}{2}\left(T_{high} + T_{low}\right).$$

Now, if the mean temperature is below 65°F we will have to supply heat on that day, and the amount of heat we supply will be proportional to the temperature difference, $65^o F - \overline{T}$. If the mean temperature is above 65°F then we will not have to supply any heat. We define the *heating degree day* (HDD) to be the product of the temperature difference times one day, that is

$$HDD \equiv \left(65^{\circ} F - \overline{T}\right)\left(1\,\text{day}\right).$$

Example 3:
On October 11 the low and high temperatures recorded for Oberlin were 45°F and 65°F respectively. On June 25 the low and high temperatures were 55°F and 77°F respectively. Calculate the *heating degree-day* values for these two dates in Oberlin.

Solution:
The mean temperature for October 11 is 55°F. Thus, the HDD value for that day is
HDD   = (65°F-55°F) (1 day)
       = 10 degree-days.
On June 25 the mean temperature was 66°F. Since this is greater than 65°F, the HDD value for that day is zero.

Heating degree day values are tabulated for each day at various locations then added up for an entire year. Maps are available (similar to weather maps which show temperatures around the US) which show annual values of heating degree-days for various locations. For the northeast Ohio area, the yearly number ranges from 6000 - 7000 heating degree-days.[6] Detailed climate information may be found at www.csuohio.edu/nws/climate/cle/lcdclejan00.html.

4.4 Annual Heat Loss through Building Envelope

Once we have calculated the total UA product for a building we may combine this with the annual number of heating degree days to calculate the expected annual heat loss through the building envelope. This is one of the major ways in which a building uses energy -- to offset the heat which flows from inside to outside through its walls during cold weather. This annual amount of energy is given by the following expression:

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[6]   See Hinrich, p. 132.

$$\Delta E = (UA)_{total} (HDD)\left(24\,\frac{hr}{day} \right).$$

In designing a building it is easy to use the number of heating degree days in a typical year. But the same formula applies for a month or even a day when the HDD figure used corresponds to these intervals of time rather than to the entire year.

Example 4:

A certain building in Oberlin, OH is shaped like a box with a flat roof. Its dimensions are 20 ft. long x 10 ft. wide x 10 ft. high. The building has now windows or doors (OK, kinda silly building). The walls have an R-value of 11 and the roof has an R-value of 19. We will ignore any heat loss through the floor. The average annual HDD figure for Oberlin is 6276 degree-day. Calculate the total UA product for the building, the peak heat load, and the expected annual energy-loss through the building envelope.

Solution:

The total UA product is given by

$(UA)_{total} = (20\ ft^2 + 20\ ft^2 + 10\ ft^2 + 10\ ft^2)/11 + 20\ ft^2/19$

$\qquad\qquad = 6.51\ Btu/hr/°F.$

The maximum heat load is then

$P_{max} = (UA)_{total}\ (T_i - T_{design}) = (6.51\ Btu/hr/°F)(65°F-5°F)$

$\qquad = 390\ Btu/hr.$

The annual energy lost this way is given by

$E = (UA)_{total}\ (HDD)\ (24\ hrs/day) = (6.51\ Btu/hr/°F)\ (6276\ °F\text{-}day)\ (24\ hrs/day)$

$\qquad = 980,000\ Btu.$

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5.   Infiltration

In any building fresh air must be brought in from the outside and heated (or cooled) to the inside temperature. In an old house the air simply leaks in/out through a variety of gaps in the skin. Typical areas of infiltration are around windows and doors. To reduce infiltration these should be plugged with caulking or insulation. In a super-insulated home, there is little or no infiltration due to a highly sealed wall. Even so, however, good health requires that fresh air be brought in on a regular basis. We have already seen how to calculate the amount of heat required to warm this air. In a modern, super-insulated building all of the outside air is brought in at one location (connected with the furnace/AC) which allows the use of a heat-exchanger. This device recaptures 60-75% of the energy of the air being exhausted and uses it to preheat the outside air.

Let us suppose that the inside and outside temperatures are T_i and T_O respectively. Cold air brought in from the outside must have its temperature raised by an amount $\Delta T = T_i - T_O$. Let c be the specific heat of air *per unit volume* (the usual specific heat is given per unit mass). Then if a volume ΔV of air is to be heated so as to raise its temperature by an amount ΔT, the energy required is

$$\Delta E = c(\Delta V)\Delta T$$.

The infiltration rate is measured in number of air changes each hour, σ. For instance, a particularly drafty old home may have its entire air changed each hour, $\sigma = 1\ \text{hr}^{-1}$. In contrast, a highly sealed new structure might have $\sigma = 0.2\ \text{hr}^{-1}$ (i. e., it takes 5 hours to completely change all the air in the structure). To calculate the energy required we need to convert this number into a volume of air. Let V be the house volume and σ be its air exchange rate. Then the volume of air that must be heated in a time ΔT is given by

$$\Delta V = \sigma V \Delta t$$.

The rate at which energy must be supplied to heat the incoming air is then give by

$$P_{air} = \frac{\Delta E}{\Delta t} = (\sigma c V)\Delta T$$.

Example 5:

Consider a 2,500 sq. ft. home having an air-exchange rate of 0.7 changes per hour. For simplicity assume the height is 8 ft. Calculate the peak heat load for heating the air assuming a heating design temperature of 5°F.

Solution:
From Table I in Chapter 2 we see that the volume specific heat of nitrogen is 1,300 $J/m^3/°C$ (the product of the mass specific heat and the density). The interior volume is
V   = (2,500 sq. ft.) (8 ft) = 20,000 cu. ft. = 570 m^3.
ΔT = (65°F-5°F) = 33°C

P_{max} = σ c V ΔT = (0.7 hr^{-1}) (1300 $J/m^3/°C$) (570 m^3) (33°C) / (1054 J/Btu)
    = 16,000 Btu/hr = 4.8 kW
Thus, in each hour, it takes 4.8 kW-hr to heat the cold air that is leaking in.

Note that it is very difficult to estimate an air infiltration rate. For typical homes the rate varies from 0.2 to 1.0 changes per hour. The best thing to do is to actually measure the infiltration rate. This can be accomplished best with a "blower door."

In the above example we calculated the peak heat load required to offset infiltration. We now calculate the annual energy required to heat this air. Recall that the heating-degree-day (HDD) adds up the product $\Delta T \Delta t$ for the entire heating season. Thus, the annual energy used to heat the air is given by

$$\Delta E_{annual} = (\sigma c V)\left(\frac{24 hr}{day}\right) HDD$$

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Example 6:
Consider, again, the 2500 sq. ft. home having an air-exchange rate of 0.7 changes per hour. Calculate the annual energy required to heat the air due to infiltration, assuming an HDD = 6276 (appropriate for Oberlin).

Solution:
ΔE $= \sigma$ c V (HDD) (24 hr/day)
$= (0.7 \text{ hr}^{-1})$ (1300 J/m^3/°C) (570 m^3) (24 hr/day) (6276 °F-day) (5°C/9°F) / (1054 J/Btu)
$= 4.3 \times 10^{10}$ J = 41 million Btu = 12,100 kW-hr.

Thus, on average we must supply 1.38 kW of heating power (continually) to offset the amount of energy being used to heat the make-up air that is being continually supplied to the home.

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6. Home Energy Audit

The annual energy used by a home for heating then includes both the heat required to offset losses through the thermal envelope and the energy required to heat the air. A home energy audit is a method for assessing the amount of energy lost in the home.