Assignment #7

PHYS-410
Fall 2013
Mr. Scofield

Announcements

Reading

Begin reading Chapter 7 of the textbook. In treating systems of N, indistinguishable particles we have, until now, failed to account for their intrinsic spin. Recall that fermions obey the *Pauli Exclusion Principle* while bosons do not. This has profound impact on the ground state of an N-particle system. We now take up this issue to develop the *Fermi-Dirac* and *Bose-Einstein* distribution functions. To do this we must first consider systems for which the number of particles N is allowed to vary. This leads us to the grand-canonical ensemble, the Gibbs factor, and the grand partition function.

With these new mathematical tools developed we apply them to some important systems: an ideal fermi gas (e.g., conduction electrons in a metal), blackbody radiation, and the Debye model for the heat capacity of a solid.

Important formulas:

Gibbs Factor: $e^{-\beta(E_\alpha - \mu N_\alpha)}$

Grand partition function: $\mathcal{Z} = \sum_\alpha e^{-\beta(E_\alpha - \mu N_\alpha)}$

F-D distribution: $f(\varepsilon) = \dfrac{1}{1+e^{\beta(\varepsilon-\mu)}}$

B-E distribution: $b(\varepsilon) = \dfrac{1}{1-e^{\beta(\varepsilon-\mu)}}$

Homework Problems (from Schroeder, unless otherwise specified)

Your solutions to the problems below are due at the beginning of class, Friday, Nov. 8.

<u>7.01</u> Show that the Fermi-Dirac function $f(\varepsilon)$ has the symmetry $f(\mu+\delta) = 1 - f(\mu-\delta)$.

Thus, the probability that a single particle energy level an amount δ above the chemical potential is occupied is equal to the probability that a level an amount δ below the chemical potential is vacant.

7.02 **Problem 7.11.** For a system of fermions at room temperature, compute the probability of a single-particle state being occupied if its energy is

- **(a)** 1 eV less than μ
- **(b)** 0.01 eV less than μ
- **(c)** equal to μ
- **(d)** 0.01 eV greater than μ
- **(e)** 1 eV greater than μ

7.03 Problem 7.19 (but for potassium instead of copper)

Problem 7.19. Each atom in a chunk of copper contributes one conduction electron. Look up the density and atomic mass of copper, and calculate the Fermi energy, the Fermi temperature, the degeneracy pressure, and the contribution of the degeneracy pressure to the bulk modulus. Is room temperature sufficiently low to treat this system as a degenerate electron gas?

7.04 **Problem 7.22.** Consider a degenerate electron gas in which essentially all of the electrons are highly relativistic ($\epsilon \gg mc^2$), so that their energies are $\epsilon = pc$ (where p is the magnitude of the momentum vector).

- **(a)** Modify the derivation given above to show that for a relativistic electron gas at zero temperature, the chemical potential (or Fermi energy) is given by $\mu = hc(3N/8\pi V)^{1/3}$.
- **(b)** Find a formula for the total energy of this system in terms of N and μ.

7.05 **Problem 7.26.** In this problem you will model helium-3 as a noninteracting Fermi gas. Although ^{3}He liquefies at low temperatures, the liquid has an unusually low density and behaves in many ways like a gas because the forces between the atoms are so weak. Helium-3 atoms are spin-1/2 fermions, because of the unpaired neutron in the nucleus.

- **(a)** Pretending that liquid ^{3}He is a noninteracting Fermi gas, calculate the
- **(b)** Calculate the heat capacity for $T \ll T_F$, and compare to the experimental result $C_V = (2.8\ \mathrm{K}^{-1})NkT$ (in the low-temperature limit). (Don't expect perfect agreement.)
- **(c)** The entropy of *solid* ^{3}He below 1 K is almost entirely due to its multiplicity of nuclear spin alignments. Sketch a graph S vs. T for liquid and solid ^{3}He at low temperature, and estimate the temperature at which the liquid and solid have the same entropy. Discuss the shape of the solid-liquid phase boundary shown in Figure 5.13.

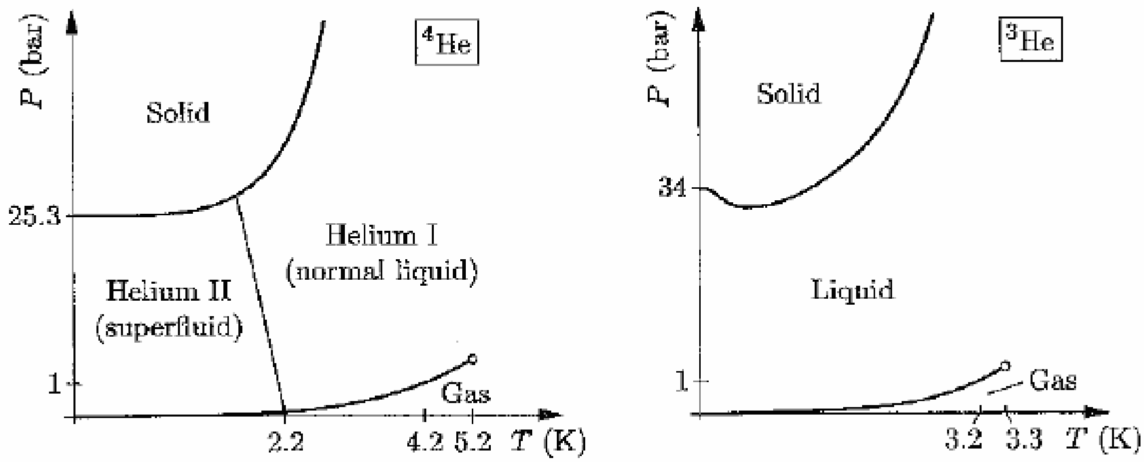

Figure 5.13. Phase diagrams of ^{4}He (left) and ^{3}He (right). Neither diagram is to scale, but qualitative relations between the diagrams are shown correctly. Not shown are the three different solid phases (crystal structures) of each isotope, or the superfluid phases of ^{3}He below 3 mK.

7.06 **Problem 7.28.** Consider a free Fermi gas in two dimensions, confined to a square area $A = L^2$.

(a) Find the Fermi energy (in terms of N and A), and show that the average energy of the particles is $\epsilon_F/2$.

(b) Derive a formula for the density of states. You should find that it is a constant, independent of ϵ.

(c) Explain how the chemical potential of this system should behave as a function of temperature, both when $kT \ll \epsilon_F$ and when T is much higher.

(d) Because $g(\epsilon)$ is a constant for this system, it is possible to carry out the integral 7.53 for the number of particles analytically. Do so, and solve for μ as a function of N. Show that the resulting formula has the expected qualitative behavior.

(e) Show that in the high-temperature limit, $kT \gg \epsilon_F$, the chemical potential of this system is the same as that of an ordinary ideal gas.