Assignment #8

PHYS-410
Fall 2013
Mr. Scofield

Announcements

Reading

Read the rest of **Chapter 7**, with particular emphasis on section 7.4 (blackbody radiation) and 7.5 (Debye theory of solids) and 7.6 (Bose condensation). The factor of ¼ that appears on the relationship between the spectral energy density inside a cavity and the spectral radiation flux leaving the cavity is derived on pp. 300-302 (for those of you with curiosity about such things).

Homework Problems (from Schroeder, unless otherwise specified)

Your solutions to the problems below are due at the beginning of class, Friday, Nov. 15

1. Density of States for blackbody radiation

Recall in lecture that we considered the electromagnetic standing wave modes in a metal cavity of volume V = L^3. The boundary conditions at the cavity walls yield standing wave modes. The wave vector and frequency are related by $\omega = ck$ with $k \equiv \sqrt{k_1^2 + k_2^2 + k_3^2}$, and the three components of the wave vector are quantized, $k_j = n_j \, \pi/L$, with n_j being any positive integer. For this problem show that the number of standing wave modes per unit volume with frequencies between $\nu \to \nu + d\nu$ is given by

$$D(\nu)\,d\nu = g\,\frac{4\pi}{c^3}\nu^2 d\nu \,,$$

where g = 2 for the two allowed photon polarizations.

2. Schroeder Problem 7.33

Problem 7.33. When the attractive forces of the ions in a crystal are taken into account, the allowed electron energies are no longer given by the simple formula 7.36; instead, the allowed energies are grouped into **bands**, separated by **gaps** where there are no allowed energies. In a **conductor** the Fermi energy lies within one of the bands; in this section we have treated the electrons in this band as "free" particles confined to a fixed volume. In an **insulator**, on the other hand, the Fermi energy lies within a gap, so that at $T = 0$ the band below the gap is completely occupied while the band above the gap is unoccupied. Because there are no empty states close in energy to those that are occupied, the electrons are "stuck in place" and the material does not conduct electricity. A **semiconductor** is an insulator in which the gap is narrow enough for a few electrons to jump across it at room temperature. Figure 7.17 shows the density of states in the vicinity of the Fermi energy for an idealized semiconductor, and defines some terminology and notation to be used in this problem.

(a) As a first approximation, let us model the density of states near the bottom of the conduction band using the same function as for a free Fermi gas, with an appropriate zero-point: $g(\epsilon) = g_0\sqrt{\epsilon - \epsilon_c}$, where g_0 is the same constant as in equation 7.51. Let us also model the density of states near the top of the valence band as a mirror image of this function. Explain why, in this approximation, the chemical potential must always lie precisely in the middle of the gap, regardless of temperature.

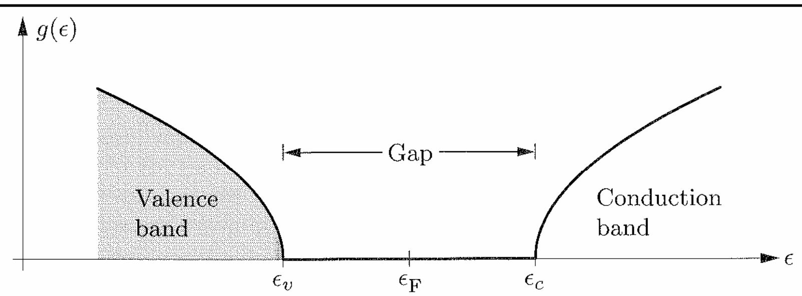


Figure 7.17. The periodic potential of a crystal lattice results in a density-of-states function consisting of "bands" (with many states) and "gaps" (with no states). For an insulator or a semiconductor, the Fermi energy lies in the middle of a gap so that at $T = 0$, the "valence band" is completely full while the "conduction band" is completely empty.

(b) Normally the width of the gap is much greater than kT. Working in this limit, derive an expression for the number of conduction electrons per unit volume, in terms of the temperature and the width of the gap.

(c) For silicon near room temperature, the gap between the valence and conduction bands is approximately 1.11 eV. Roughly how many conduction electrons are there in a cubic centimeter of silicon at room temperature? How does this compare to the number of conduction electrons in a similar amount of copper?

(d) Explain why a semiconductor conducts electricity much better at higher temperatures. Back up your explanation with some numbers. (Ordinary conductors like copper, on the other hand, conduct better at *low* temperatures.)

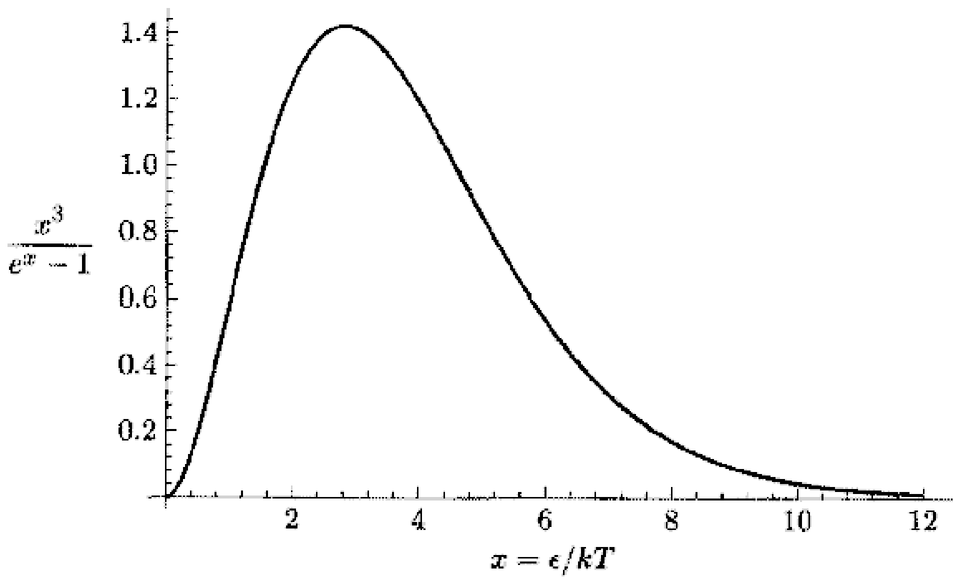
(e) Very roughly, how wide would the gap between the valence and conduction bands have to be in order to consider a material an insulator rather than a semiconductor?

3. Schroeder Problem 7.37

Prove that the peak in the Planck energy (or frequency) spectrum occurs at $x \approx 2.82$, where $x \equiv hf/k_BT$. For those who like a challenge, consider the Planck wavelength spectrum. At what wavelength does it peak?

4. Schroeder Problem 7.38

Problem 7.38. It's not obvious from Figure 7.19 how the Planck spectrum changes as a function of temperature. To examine the temperature dependence, make a quantitative plot of the function $u(\epsilon)$ for $T = 3000$ K and $T = 6000$ K (both on the same graph). Label the horizontal axis in electron-volts.



5. Schroeder Problem 7.43

Problem 7.43. At the surface of the sun, the temperature is approximately 5800 K.

(a) How much energy is contained in the electromagnetic radiation filling a cubic meter of space at the sun's surface?

(b) Sketch the spectrum of this radiation as a function of photon energy. Mark the region of the spectrum that corresponds to visible wavelengths, between 400 nm and 700 nm.

(c) What fraction of the energy is in the visible portion of the spectrum? (Hint: Do the integral numerically.)

6. Schroeder Problem 7.46

Problem 7.46. Sometimes it is useful to know the free energy of a photon gas.

(a) Calculate the (Helmholtz) free energy directly from the definition $F = U - TS$. (Express the answer in terms of T and V.)

(b) Check the formula $S = -(\partial F/\partial T)_V$ for this system.

(c) Differentiate F with respect to V to obtain the pressure of a photon gas. Check that your result agrees with that of the previous problem.

(d) A more interesting way to calculate F is to apply the formula $F = -kT \ln Z$ separately to each mode (that is, each effective oscillator), then sum over all modes. Carry out this calculation, to obtain

$$F = 8\pi V \frac{(kT)^4}{(hc)^3} \int_0^\infty x^2 \ln(1 - e^{-x})\, dx.$$

Integrate by parts, and check that your answer agrees with part (a).