Oberlin College Physics 110, Fall 2011

Model Solutions to Sample Final Exam

Additional problem 27: *Cannon shot*
Because I’m concerned about speeds and distances, but not times, the central equation will likely be

\[ v^2 = v_0^2 + 2a_0(x - x_0). \]

In our case \( v_0 = 0 \), \( x - x_0 = L \), the length of the cannon, so

\[ a_0 = \frac{v^2}{2L}. \]

To find the time, use

\[ v = v_0 + a_0t \]

or

\[ T = \frac{v}{a_0} = \frac{2L}{v}. \]

Plugging in the given numbers results in a time of 0.6600 s. (Note four significant figures.)

Additional problem 76: *Spring gun*
Solved in the notes.

Additional problem 90: *Train latch*
Let \( M_f \) and \( M_c \) represent the masses of the freight car and the caboose. Let \( v_i \) represent the initial velocity of the freight car and \( v_f \) represent the final velocity of the latched combination.

- Momentum conservation: \( M_f v_i = (M_f + M_c) v_f \).
- Kinetic energy loss: \( 0.66(\frac{1}{2} M_f v_i^2) = \frac{1}{2} (M_f + M_c) v_f^2 \).

Square the first equation, and divide that squared equation by the second equation to obtain

\[ \frac{1}{0.66} M_f = M_f + M_c \quad \text{or} \quad M_c = \frac{0.37}{0.66} M_f. \]

Plugging in the given weight of the freight car, the caboose has weight 24 tons.

HRW problem 9-17: *A dog on a boat*
The location of the center of mass is

\[ x_{cm} = \frac{x_B m_B + x_D m_D}{m_B + m_D} \]

where \( x_B \) is the location of the CM of the boat, \( m_B \) the mass of the boat, and similarly for the dog.

There are no external forces during the dog’s walk, so \( \Delta x_{cm} = 0 \), whence

\[ \Delta x_B m_B = -\Delta x_D m_D. \]

Now, \( \Delta x_D = -2.4 \text{ m} + \Delta x_B \) so

\[ \Delta x_B = (2.4 \text{ m}) \left( \frac{m_B}{m_B + m_D} \right) = 0.48 \text{ m}. \]
So the distance from the dog to shore is 

\[ 6.1 \text{ m} - 2.4 \text{ m} + 0.48 \text{ m} = 4.2 \text{ m}. \]

**Relativity problem 2: Muon lifetime**
Classically, without time dilation:

\[ \text{distance traveled} = \text{speed} \times \text{time} = (0.83 \text{ c}) \times (2.2 \mu\text{s}) = 550 \text{ m} \]

Correctly, with time dilation:

\[ T_0 = \text{time ticked off by muon between production and decay} = 2.2 \mu\text{s} \]
\[ T = \text{time elapsed in lab frame between production and decay} = \frac{T_0}{\sqrt{1 - (V/c)^2}} = \frac{2.2 \mu\text{s}}{\sqrt{1 - (0.83)^2}} = 3.9 \mu\text{s} \]

\[ \text{distance traveled in lab frame} = \text{speed in lab frame} \times \text{time elapsed in lab frame} = (0.83 \text{ c}) \times (3.9 \mu\text{s}) = 980 \text{ m} \]

**Relativity problem 8: Time travel**
Ivan has aged \( T_0 = 2 \text{ years} \) whereas time \( T = 12 \text{ years} \) has elapsed, so

\[ T = \frac{T_0}{\sqrt{1 - (V/c)^2}} \]
\[ \sqrt{1 - (V/c)^2} = \frac{T_0}{T} = 1/6 \]
\[ 1 - (V/c)^2 = 1/36 \]
\[ (V/c)^2 = 35/36 \]
\[ V = \sqrt{35/36} c = 0.986 c \]

**Relativity problem 11: Two events**
The Lorentz transform says that

\[ \Delta t' = \frac{\Delta t - V \Delta x/c^2}{\sqrt{1 - (V/c)^2}}. \]

So if \( \Delta t' = 0 \), we have

\[ 0 = \Delta t - V \Delta x/c^2 \]
\[ V = (\Delta t/\Delta x)c^2 = (6 \text{ nan}/14 \text{ ft})(1 \text{ ft}/\text{nan})c = \frac{3}{7} c. \]

**Relativity problem 18: Relativistic energy: a new proposal**
If this proposal were correct, then the non-relativistic limit of energy would be

\[ E = \frac{mc^2}{\sqrt{1 - (v/c)^2}} \approx mc^2 [1 - \frac{1}{2} (-(v/c)^4)] = mc^2 + \frac{1}{2}mv^4/c^2. \]

That is, classical kinetic energy would be, not \( \frac{1}{2}mv^2 \), but \( \frac{1}{2}mv^4/c^2 \). Clearly wrong.