## An island in the stream

Objective: Find magnetic field $\vec{B}$ at point $C$ :

A. Hard way. What is $\vec{B}$ at $C$ due to current in the left-hand straight wire?


It is zero, because a moving charge carrier produces no $\vec{B}$ on the velocity axis.
Similarly, there is no $\vec{B}$ at $C$ due to the current in the right-hand straight wire.
What is $\vec{B}$ at $C$ due to current in the upper half circle?


The $\vec{B}$ due to a full circle would have magnitude $\frac{\mu_{0}(i / 2)}{2 R_{\vec{B}}}$, direction into the page (magnitude from LSM equation 12.17, direction from right hand rule), so the $\vec{B}$ due to the upper half circle would be half this: magnitude $\frac{\mu_{0} i}{8 R}$, direction into the page.

What is $\vec{B}$ at $C$ due to current in the lower half circle? Following the reasoning above, it would have the same magnitude but the opposite direction, namely out of the page.

Thus the total magnetic field at $C$ vanishes.
B. Easy way. Rotate the figure by $180^{\circ}$ about the wire axis (shown as a dotted line) and it comes back to itself.


Thus, the magnetic field at $C$ must be the same when rotated as described. The only such vector is one that points along the rotation axis. But it's clear from the Biot-Savart law that $\vec{B}$ must point perpendicular to the page. So $\vec{B}$ at $C$ must be zero.
(This argument makes it clear that $\vec{B}=0$ not only at point $C$, but at any point on the axis.)
Grading: This is a hard one for partial credit. Any correct argument earns full credit.

