Clock of charge



The contribution to the total \vec{E} at center from the charge -1q points toward one o'clock and has magnitude

$$E_1 \equiv \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}.$$

The contribution from the charge -2q points toward two o'clock and has magnitude $2E_1$, and so forth. (See left half of figure.)

It is a formidable problem to add all twelve of these arrows if we first add the contribution from -1q to the contribution from -2q, then add that sum to the contribution from -3q, etc. But it is rather easy to add two vectors pointing in the same direction, so we'll add the contribution from -1q to the contribution from -7q, and the contribution from -2q to the contribution from -8q, etc., resulting in six arrows (see right half of figure) that still need to be summed. Each of these six arrows has magnitude $6E_1$. (This is an application of the ancient problem-solving strategy "divide and conquer".)

Summing these six arrows is formidable if you add them in sequence, but note that the nine o'clock and ten o'clock arrows sum to an arrow in the direction of 9:30. So do the eight o'clock and eleven o'clock arrows. So do the seven o'clock and twelve o'clock arrows.

Thus the total \vec{E} field vector at the center points toward 9:30.

Grading: 2 points for a figure

2 points for realizing you must use Coulomb's law

3 points for the strategy of adding by pairs at the opposite side of the dial

2 points for strategy of adding the resulting six vectors by pairs

1 point for result