Quantum Mechanics

Sample Exam for First Examination

1 **State representations**

Write the column matrix that represents the state $|z-\rangle$ in the basis $\{|\theta+\rangle, |\theta-\rangle\}$, as a function of the angle θ.

2 **Photon polarization**

A photon linearly polarized at an angle θ to the vertical is said to be in state $|\theta\rangle$. Special cases are $|0^\circ\rangle = |x\rangle$ and $|90^\circ\rangle = |y\rangle$. Express the state $|\theta\rangle$ as a linear combination of states $|x\rangle$ and $|y\rangle$ by finding the functions $a(\theta)$ and $b(\theta)$ such that

$$|\theta\rangle = a(\theta)|x\rangle + b(\theta)|y\rangle.$$

I encourage you to use the results from section 3.3 of *The Physics of QM*.

3 **Matrix algebra**

Find the eigenvalues and corresponding (normalized) eigenvectors of the three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

4 **Math!**

Suppose $\hat{A}$ represents a linear operator. Show that if $(a, \hat{A}a)$ is real for all vectors a, then $(b, \hat{A}c) = (c, \hat{A}b)^*$ for all vectors b and c. (That is, $\hat{A}$ is Hermitian.) (*Clue:* Employ the hypothesis with $a = b + c$ and $a = b + ic$.)

What materials (books, notes, web sites, etc.) did you consult while taking this exam?