Ground State of the Simple Harmonic Oscillator

From the discussion in the problem statement,

\[ \text{P.E.} = \frac{m\omega^2}{8} d^2, \quad \text{K.E.} = \frac{\hbar^2}{32m} \frac{1}{d^2}, \]

and the total energy is

\[ E(d) = \frac{m\omega^2}{8} d^2 + \frac{\hbar^2}{32m} \frac{1}{d^2}. \]

The minimum falls at \( d_0 \) where \( E'(d) = 0 \) so

\[ \frac{m\omega^2}{4} d_0 - \frac{\hbar^2}{16m} \frac{1}{d_0^3} = 0 \]

\[ d_0^2 = \frac{\hbar}{2m}\omega \]

whence

\[ E(d_0) = \frac{1}{8}\hbar\omega. \]

This estimate for the ground state energy is four times too small, but on the other hand it’s considerably easier to find than the true ground state energy!

Note that if \( \hbar \to 0 \), the P.E. curve does not change, but the K.E. curve moves left and shrinks down into the corner. The P.E. curve dominates, so the minimum energy comes at the minimum P.E., namely \( d_0 = 0 \).