Model Solutions Concerning Interference

Tilted analyzer loop

paths blocked	prob. of passing input to intermediate	intermediate condition	prob. of passing intermediate to output	overall probability
none	1	$\mu_z = +\mu_B$	0	0
b	$\cos^2(\theta/2)$	$\mu_\theta = +\mu_B$	$\sin^2(-\theta/2)$	$\cos^2(\theta/2)\sin^2(\theta/2)$
a	$\sin^2(\theta/2)$	$\mu_\theta = -\mu_B$	$\sin^2[(180^\circ - \theta)/2]$	$\sin^2(\theta/2)\cos^2(\theta/2)$

These results give the proper special cases for $\theta = 90^\circ$ and $\theta = 0^\circ$.

The above is a perfectly correct solution. If you like playing with the trig identity

$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

you will find that these two equal probabilities can also be written as

$$\tfrac{1}{4}\sin^2(\theta).$$

⟦*Grading:* 1 point for free; 3 points for "no paths blocked"; for "path b blocked", 1 point for "probability input to intermediate" and 2 points for "probability intermediate to output"; for "path a blocked", 1 point for "probability input to intermediate" and 2 points for "probability intermediate to output".⟧

Three analyzer loops

Only two principles are needed to solve this problem: (a) If neither path of an interferometer is blocked, then all the atoms emerge, and they emerge in the same condition that they entered. (b) If one path of an interferometer is blocked, then it acts just like a Stern-Gerlach analyzer of the appropriate orientation.

Suppose 1000 atoms with $\mu_z = +\mu_B$ enter this contraption. Then:

paths blocked	number between 1 and 2	condition between 1 and 2	number between 2 and 3	condition between 2 and 3	number at output	condition at output
none	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$
3a	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$	500	$\mu_x = -\mu_B$
3b	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$	500	$\mu_x = +\mu_B$
2a	1000	$\mu_z = +\mu_B$	0	$\sim$	0	$\sim$
2b	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$	1000	$\mu_z = +\mu_B$
1b	500	$\mu_x = +\mu_B$	500	$\mu_x = +\mu_B$	500	$\mu_x = +\mu_B$
2a and 3b	1000	$\mu_z = +\mu_B$	0	$\sim$	0	$\sim$
1b and 3b	500	$\mu_x = +\mu_B$	500	$\mu_x = +\mu_B$	500	$\mu_x = +\mu_B$
1b and 3a	500	$\mu_x = +\mu_B$	500	$\mu_x = +\mu_B$	0	$\sim$
1b and 3a and 2a	500	$\mu_x = +\mu_B$	250	$\mu_z = -\mu_B$	125	$\mu_x = -\mu_B$

⟦*Grading:* 1 point for free; 1 point for final result of each of the nine cases starting at 3a (the "none" case is not required). For this problem (and this problem only!) grade for the final answer ("number at output" or "probability of going from input to output") and not for the explanation, as the explanation is so hard to follow.⟧

Bomb-testing interferometer

(a) If the bomb trigger is good, then it is possible to tell which path the atom takes so the atom *does* take one path or the other. With probability $\frac{1}{2}$ (2 points) it takes path a and sets off the bomb. With probability $\frac{1}{2}$ (2 points) it takes path b.

(b) If the atom takes path b, then it exits the interferometer and enters the analyzer in condition $\mu_x = -\mu_B$ (2 points). Such an atom has probability $\frac{1}{2}$ (2 points) of exiting the analyzer through the + port and probability $\frac{1}{2}$ (2 points) of exiting the analyzer through the − port.

⟦*Grading:* Each of the five answers in worth 2 points, as indicated in the model solution: 1 point for the answer, 1 point for the explanation. Explanations may be skeletal, as in the above, but they need to exist. An answer of "$\frac{1}{2}$, $\frac{1}{2}$, $\mu_x = -\mu_B$, $\frac{1}{2}$, $\frac{1}{2}$" earns five points total.⟧