Positronium

The Coulomb problem

$$\left[-\frac{\hbar^2}{2M}\nabla^2 - \frac{k}{r}\right]\eta_n(\vec{r}) = E_n\eta_n(\vec{r})$$

has eigenenergies

$$E_n = -\frac{k^2M/2\hbar^2}{n^2} \qquad n = 1, 2, 3, \ldots.$$

For the hydrogen atom problem

$$M \approx m_e \quad \text{and} \quad k = \frac{e^2}{4\pi\epsilon_0}$$

so

$$E_n \approx -\frac{\text{Ry}}{n^2} \quad \text{where} \quad \text{Ry} = \frac{(e^2/4\pi\epsilon_0)^2 m_e}{2\hbar^2} \approx 13.6 \text{ eV}.$$

For the positronium "atom" problem the reduced mass M is

$$M = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2} \quad \text{and} \quad k = \frac{e^2}{4\pi\epsilon_0}$$

so

$$E_n = -\frac{\frac{1}{2}\text{Ry}}{n^2}.$$

The bound state of an electron and a positron is unstable: the electron and positron soon annihilate into a pair of gamma rays. This Coulomb model for positronium does not take into account the forces that ultimately cause this annihilation, just as the Coulomb model for hydrogen does not take into account collisions with other atoms, or relativity, or quantum electrodynamics. In both cases, the models are imperfect yet useful. Indeed, that's the case throughout science!