Work in terms of p and V

For any quasistatic change, the dissipative work is zero so

$$\text{work} = \text{configuration work} = -\int_{\text{initial}}^{\text{final}} p(V)\, dV.$$

The question is "How does this integral relate to the area under the curve?"

(a.) This path steps always to the right, so the integral equals the area under the curve, and the work is the negative of the area under the curve.

However, if the path were traversed in the opposite direction, the integral would be the negative of the area under the curve so the work would be the positive of the area under the curve.

(b.) For the part of the path stepping to the right, up to that rightmost volume, the work is the negative of the area under the curve. But then when the path steps to the left, the work is the positive of the area under that lower curve. So this work cancels some of the negative work. The work done is the negative of the area under the curve from V_i to V_f, PLUS the negative of the area between the two branches right of V_f.

(c.) This is just the limit of case (b) when the final point (V_f, p_f) moves all the way back to the initial point. The work done is the negative of the area enclosed within the curve.

If the path were traversed counterclockwise, the work done would be the positive of the area enclosed within the curve.