

# How Did We Find Out About Quantum Mechanics?

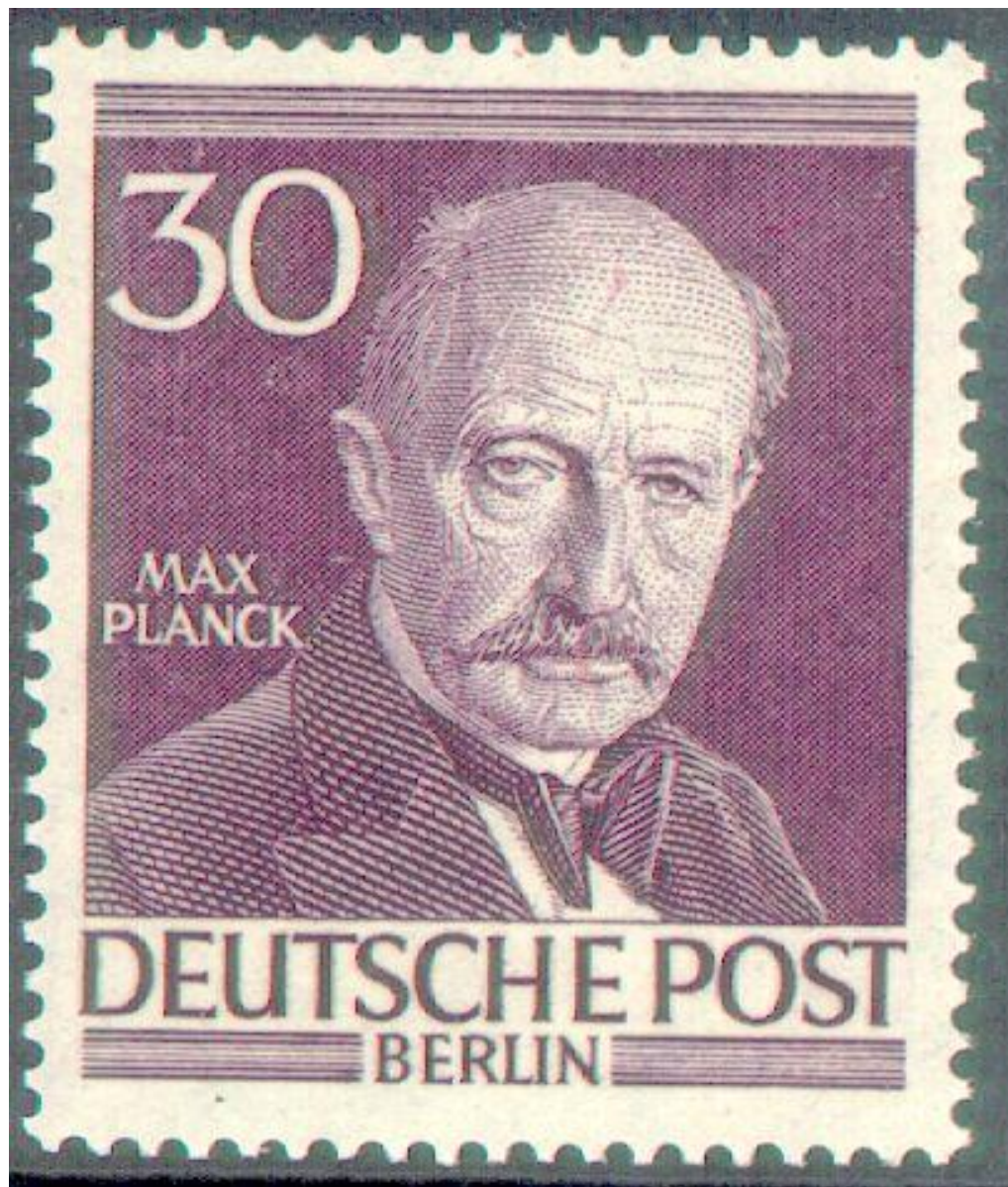
Dan Styer  
Department of Physics and Astronomy  
Oberlin College





Max Planck (age 42 in 1900)









Albert Einstein (age 26 in 1905)

Einstein's four 1905 papers:

Quantum mechanics of light (heuristic photoelectric effect)

Brownian motion (atoms exist!)

On the Electrodynamics of Moving Bodies

Does the Inertia of a Body Depend on its Energy Content?  
( $E = mc^2$ )

Einstein's four 1905 papers:

Quantum mechanics of light (heuristic photoelectric effect)  
“is very revolutionary”

Brownian motion (atoms exist!)

On the Electrodynamics of Moving Bodies  
“a modification of the theory of space and time”

Does the Inertia of a Body Depend on its Energy Content?  
( $E = mc^2$ )







Niels Bohr (age 28 in 1913)







NIELS BOHRS ATOMTEORI  
1913-1963



$$h\nu = \epsilon_2 - \epsilon_1$$

DANMARK



60

V. BANG del.

CZ. SLANIA sc.

172600A



00972A

500

500

FEM HUNDREDE  
KRONER

DANMARKS NATIONALBANK

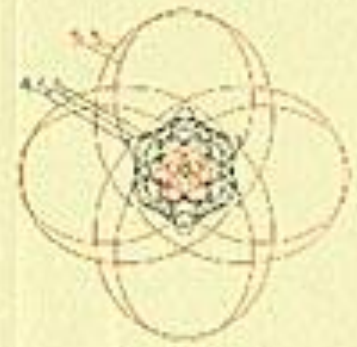
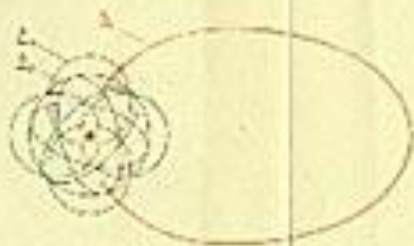
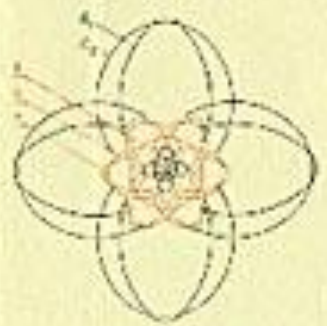
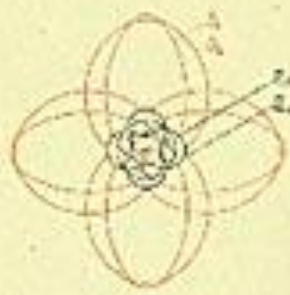


*Torben Nielsen*

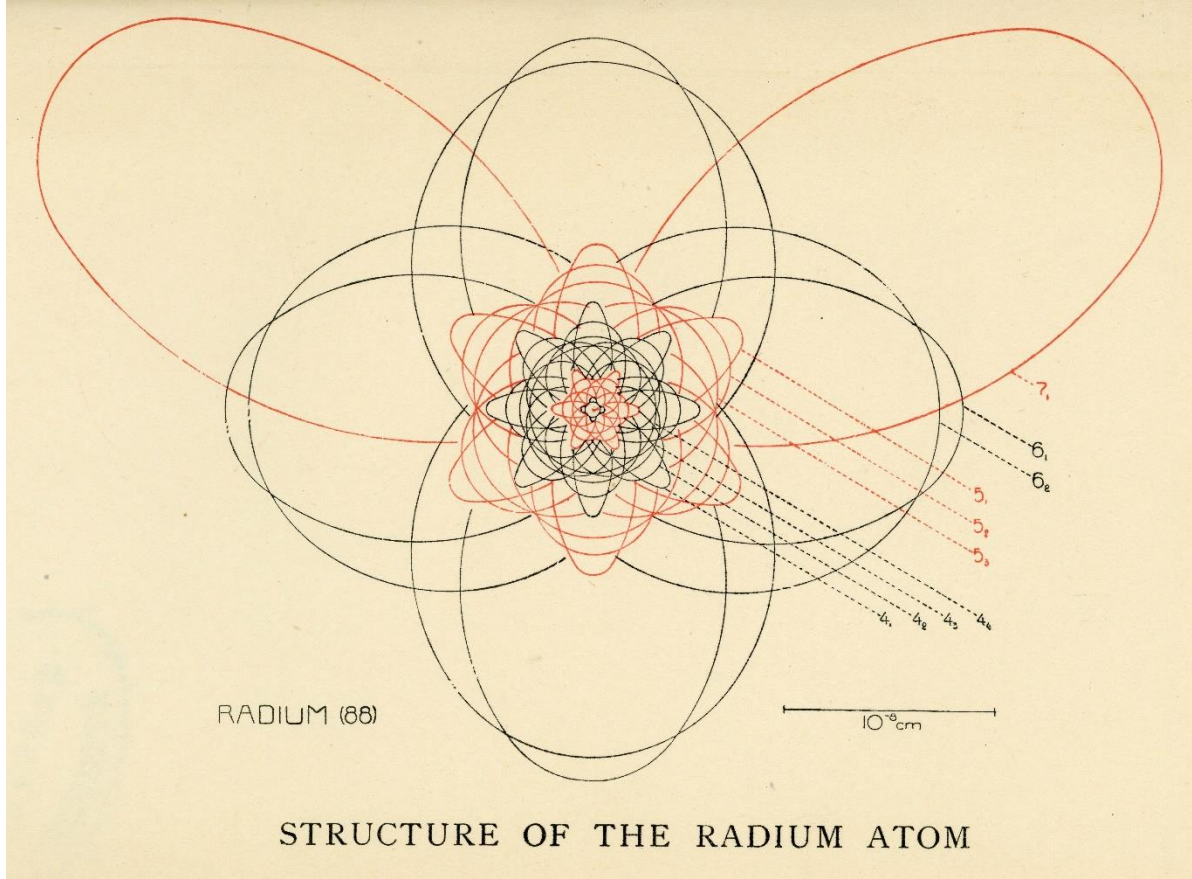
*Bohr*



MAIN LINES OF THE ATOMIC STRUCTURE  
OF SOME ELEMENTS







RADIUM (88)

$10^{-8}$  cm

STRUCTURE OF THE RADIUM ATOM



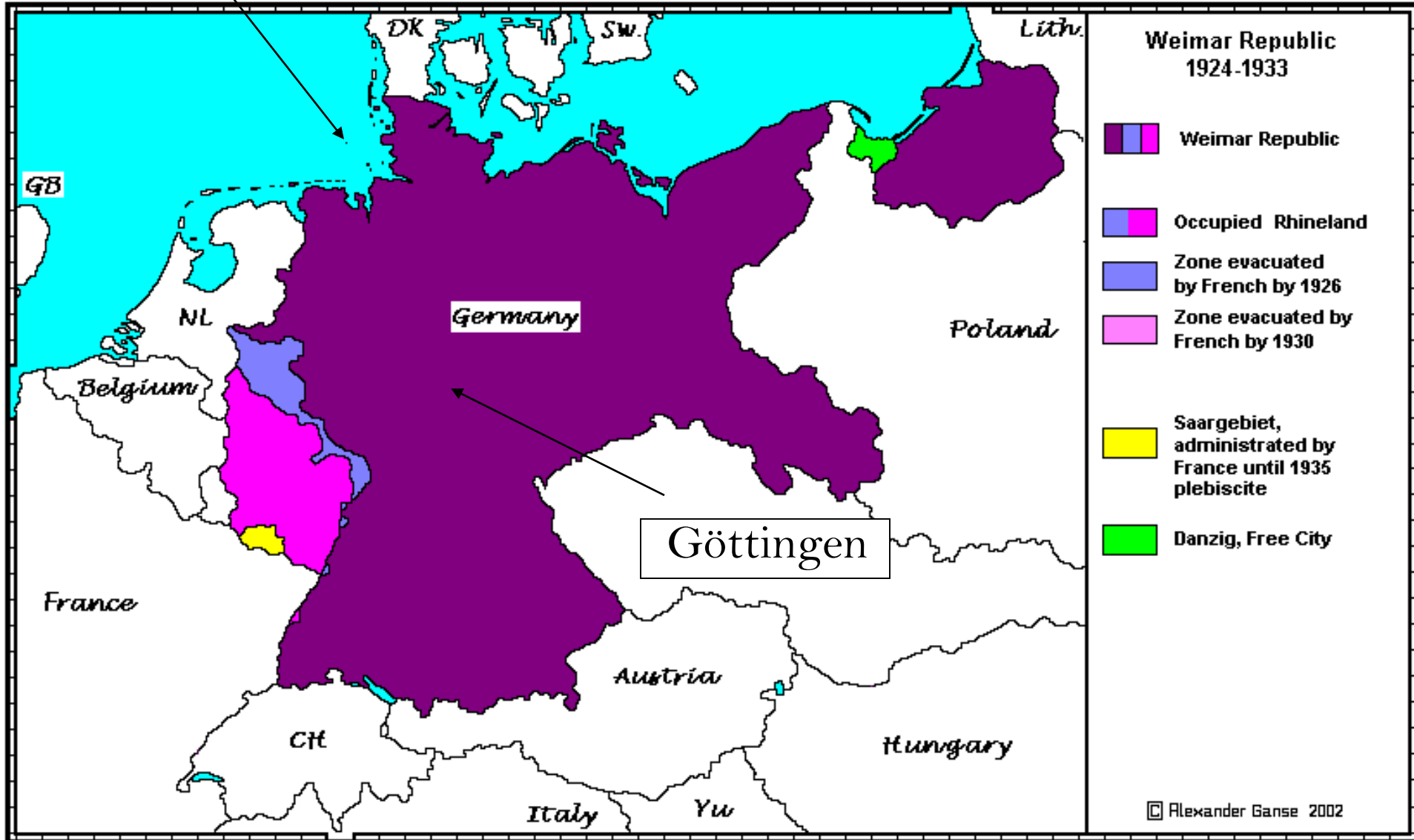
Werner Heisenberg (age 24 in 1925)





Werner Heisenberg (age 24 in 1925)

Helgoland















Wolfgang Pauli (age 25 in 1925)



Louise de Broglie (age 31 in 1923)



Louise de Broglie

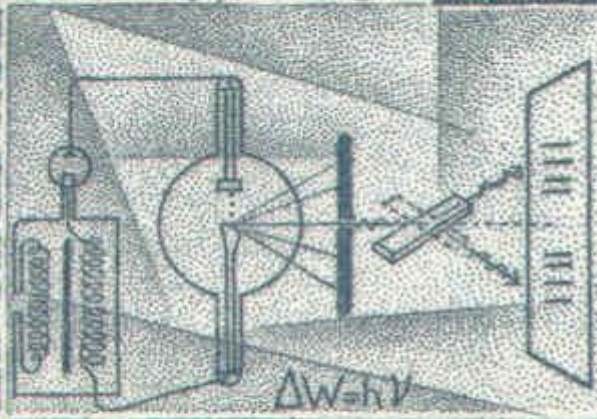


FRANCAISE

POSTES

1970

REPUBLIQUE  
SERVIR



MAURICE  
DE BROGLIE  
1875-1960

0,40+0,10



Erwin Schrödinger (age 38 in 1925)



Arosa, Switzerland





Arosa, Switzerland



Arosa, Switzerland



Erwin Schrödinger (age 38 in 1925)



Erwin Schrödinger





Crater Schrödinger

Matrix Mechanics (Heisenberg, Born)

versus

Wave Mechanics (Schrödinger)

Matrix and Wave Mechanics  
proven equivalent in 1926  
by Schrödinger, Pauli, and Carl Eckert (age 24)

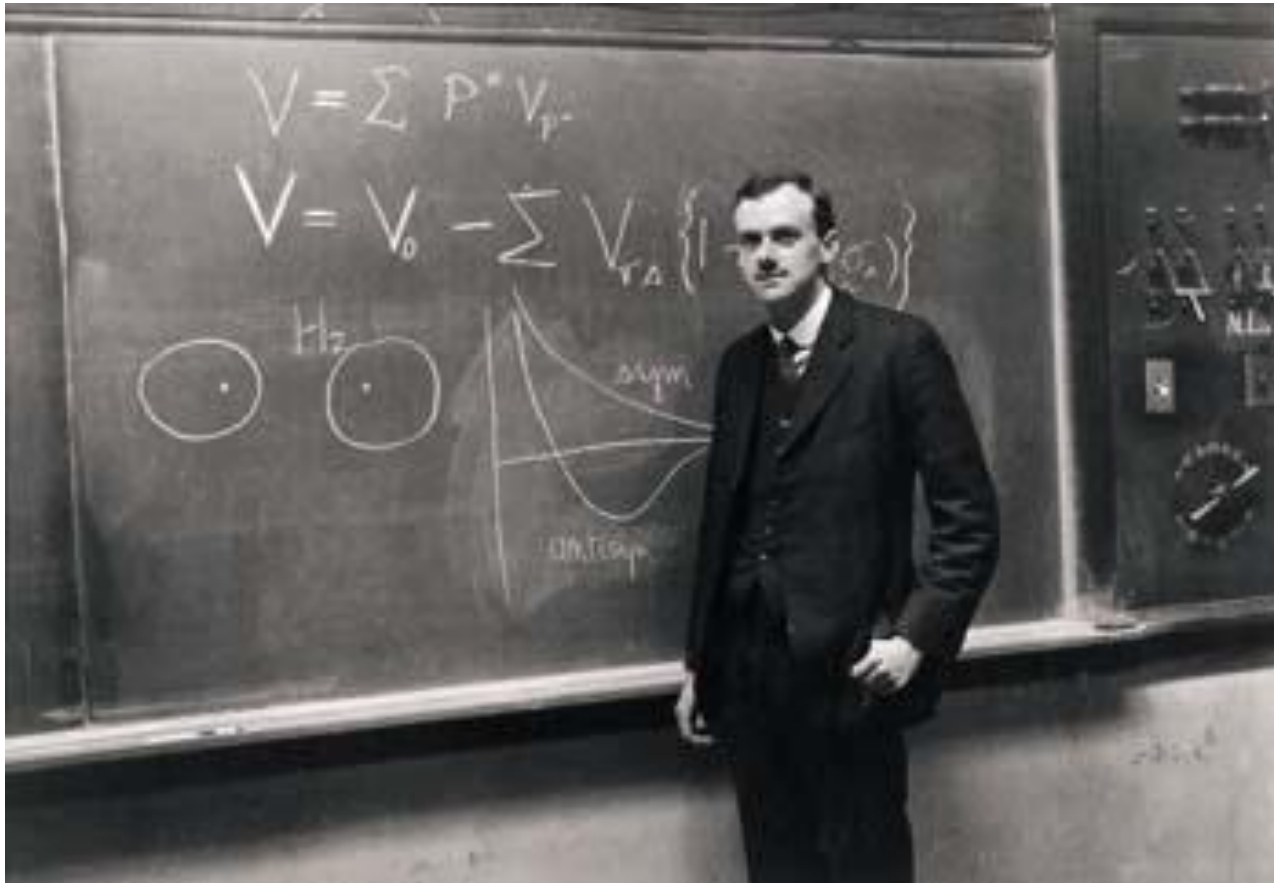


Carl Eckert (photo 1948)





Paul A. M. Dirac (age 24 in 1926)



Paul A. M. Dirac



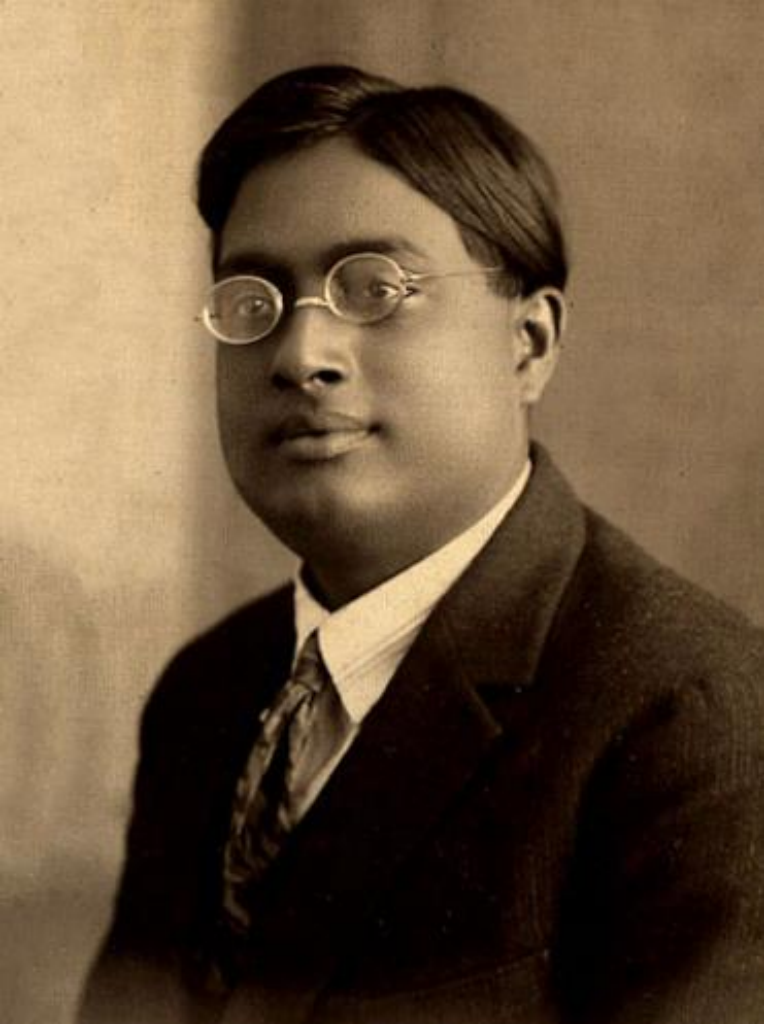
Westminster Abbey



Satyendra Bose

1924-25: quantum mechanics  
applied to large number of  
particles (with A. Einstein)

(age 31 in 1925)



## Satyendra Bose

1925: predicted a new phase of matter – solid, liquid, gas, and “Bose condensate”

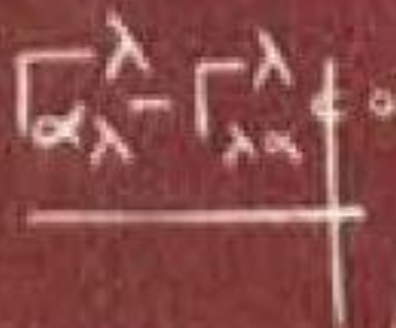




SATYENDRA NATH BOSE

सत्येन्द्र नाथ बोस

100



भारत INDIA

1994

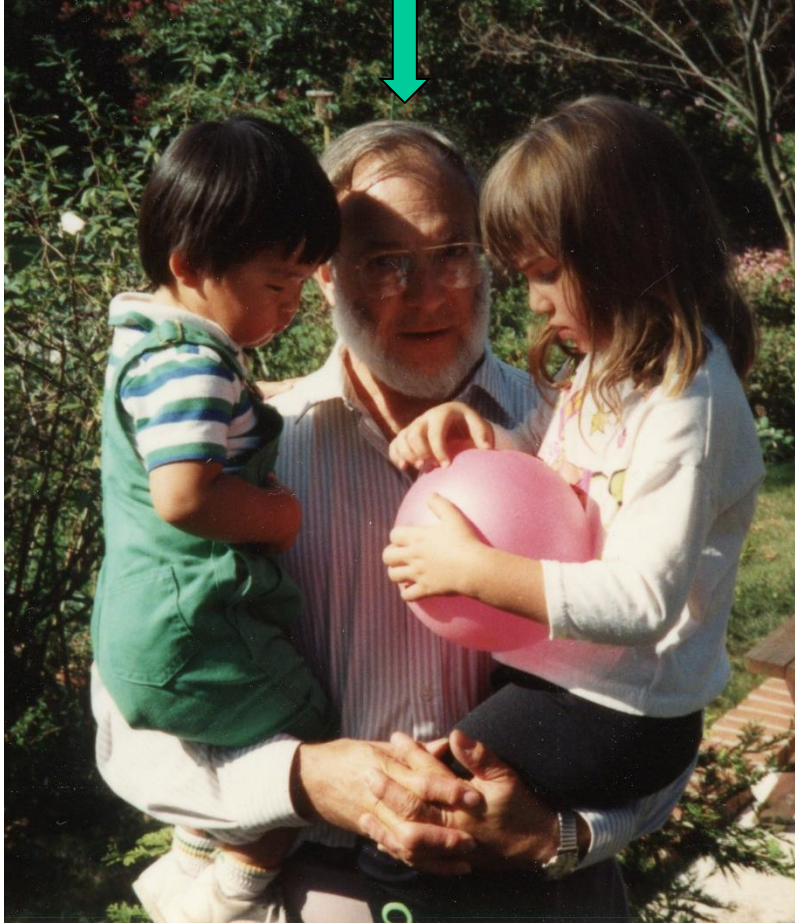




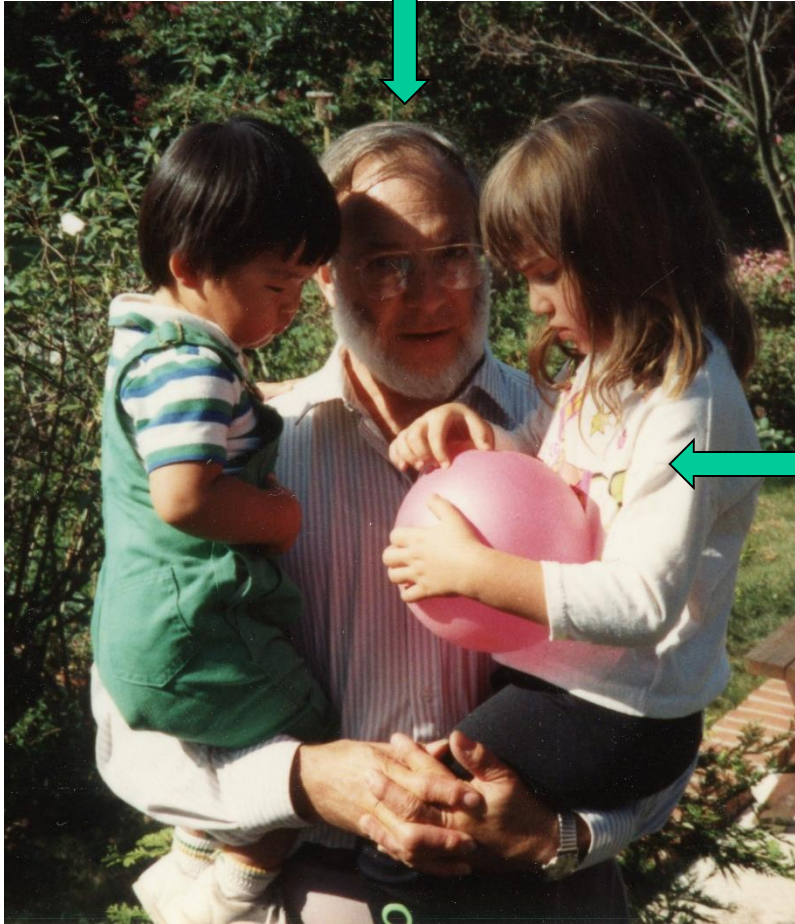
Michael Fisher

1970s: extended this work

Michael Fisher



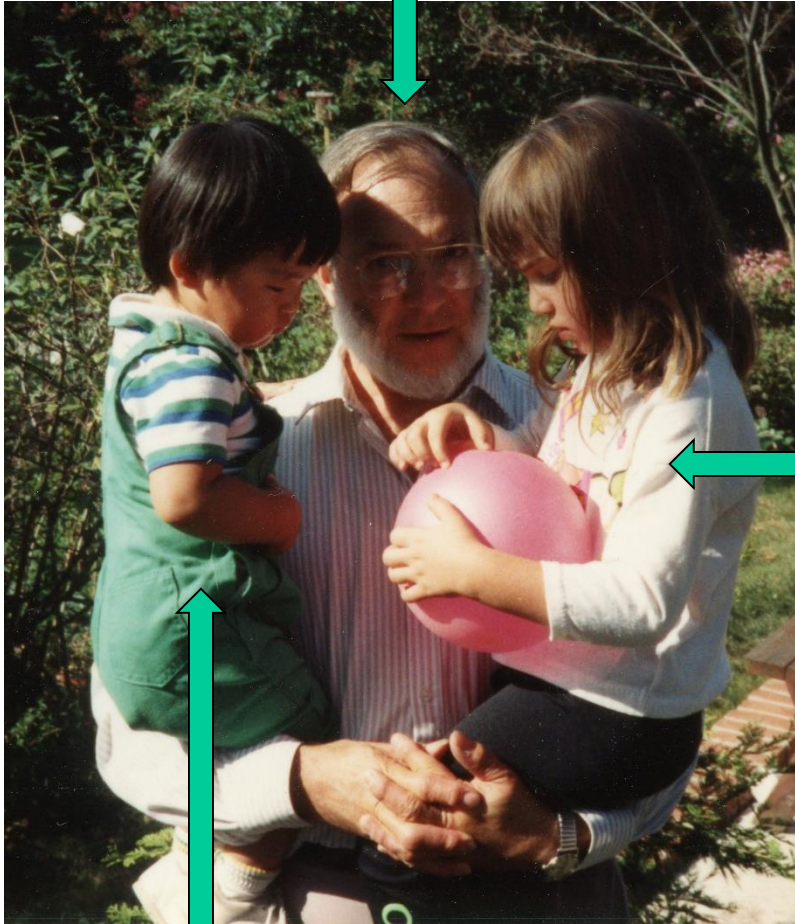
Michael Fisher



Michael Fisher's  
granddaughter



Michael Fisher



Michael Fisher's  
granddaughter

My son,  
Gregory Michael Styer

# 2 Groups of Physicists Produce Matter That Einstein Postulated

By MALCOLM W. BROWNE

By chilling a cloud of atoms to a temperature barely above absolute zero, scientists at a Colorado laboratory have at last created a bizarre type of matter that had eluded experimenters ever since its potential existence was postulated by Albert Einstein 70 years ago.

The creation of this Bose-Einstein condensate — named for Einstein and the Indian theorist Satyendra Nath Bose — was hailed yesterday as the basis of a new field of research expected to explain some fundamental mysteries of atomic physics.

A Texas group later produced similar results. The achievement should allow physicists to peer directly into the realm of the ultrasmall.

In a comment being published today by the journal *Science*, Dr. Keith Burnett, a physicist at Oxford University in England, said, "The term Holy Grail seems quite appropriate, given the singular importance of this discovery."

Details of the achievement were disclosed in a technical paper published by *Science* and at a news conference yesterday in Boulder, Colo.

Separately, a research team led

that it had independently created a Bose-Einstein condensate made of atoms different from those used in the Colorado laboratory and using a somewhat different method.

A Bose-Einstein condensate is a gas of atoms that have been so chilled that their normal motion is virtually halted. In this almost stationary condition, the wavelengths of individual atoms — the dimensions that define the regions in which they may be found — grow to relatively enormous size, overlapping each other and merging into a kind of super atom. This merged atom, despite growing to a range of sizes typical of those of bacteria, obeys the rules of quantum mechanics, the physics of the ultrasmall.

The creation of this unique state of matter by the Colorado group occurred on June 5, when a microscopic blob of Bose-Einstein condensate abruptly appeared in the glass vessel containing super-cold rubidium atoms. The condensate was de-

---

Continued on Page A14, Column 4

---



THE NEW YORK TIMES  
is available for home or office

Experimental  
verification!

1995

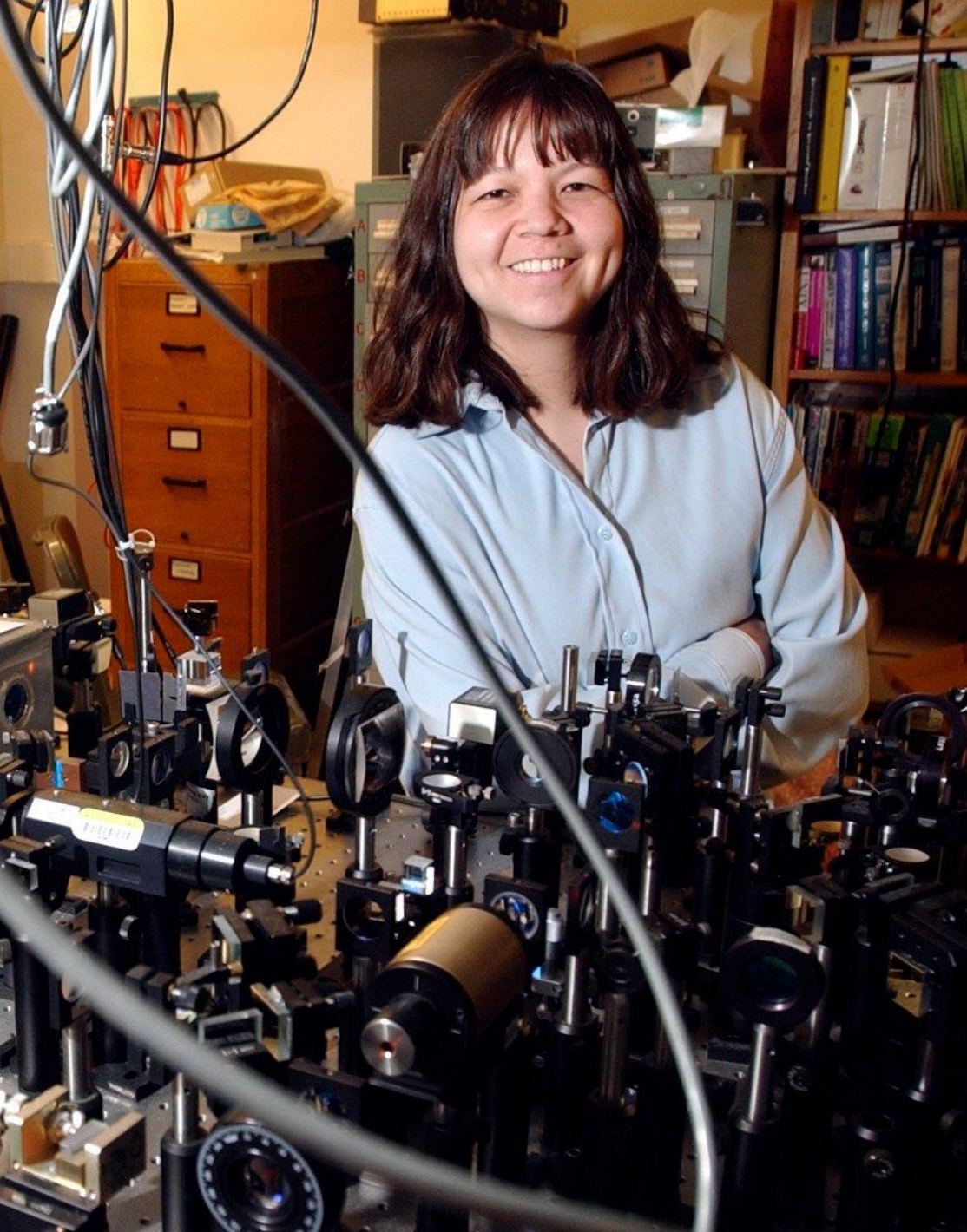


# Experimental verification!

1995

by Carl Wieman  
and Eric Cornell





Experimental  
verification!

extended to  
“fermionic  
condensate”  
by Deborah Jin  
in 2003



Linus Pauling (age 30 in 1931)





Linus Pauling and family

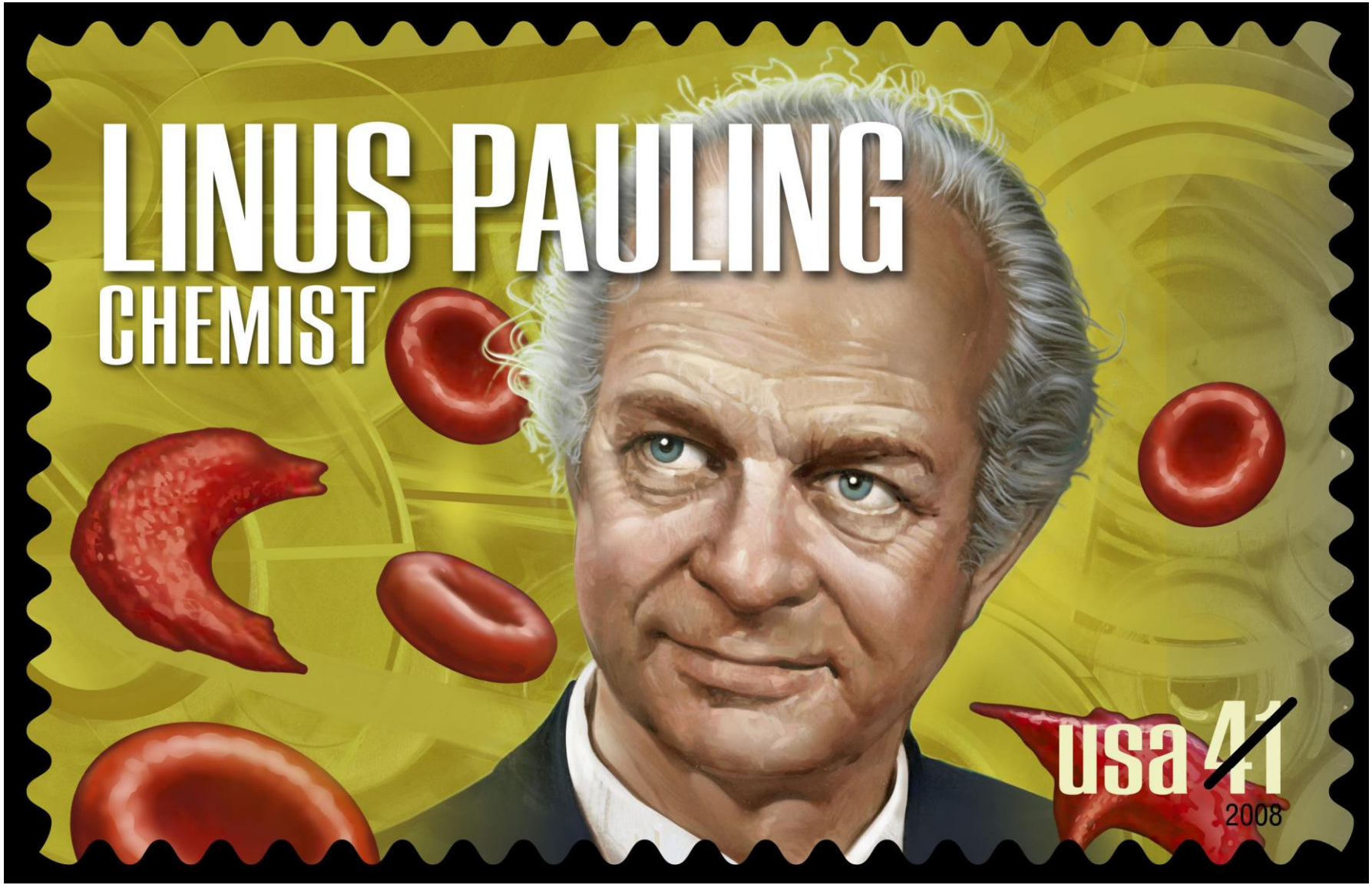


Linus Pauling



# LINUS PAULING

CHEMIST



usa 41  
2008

Linus Pauling  
gave his name to  
Linus Torvalds  
who gave his name to  
Linux





Maria Goeppert Mayer  
nuclear shell theory, 1950



2011

~~FOREVER~~

USA



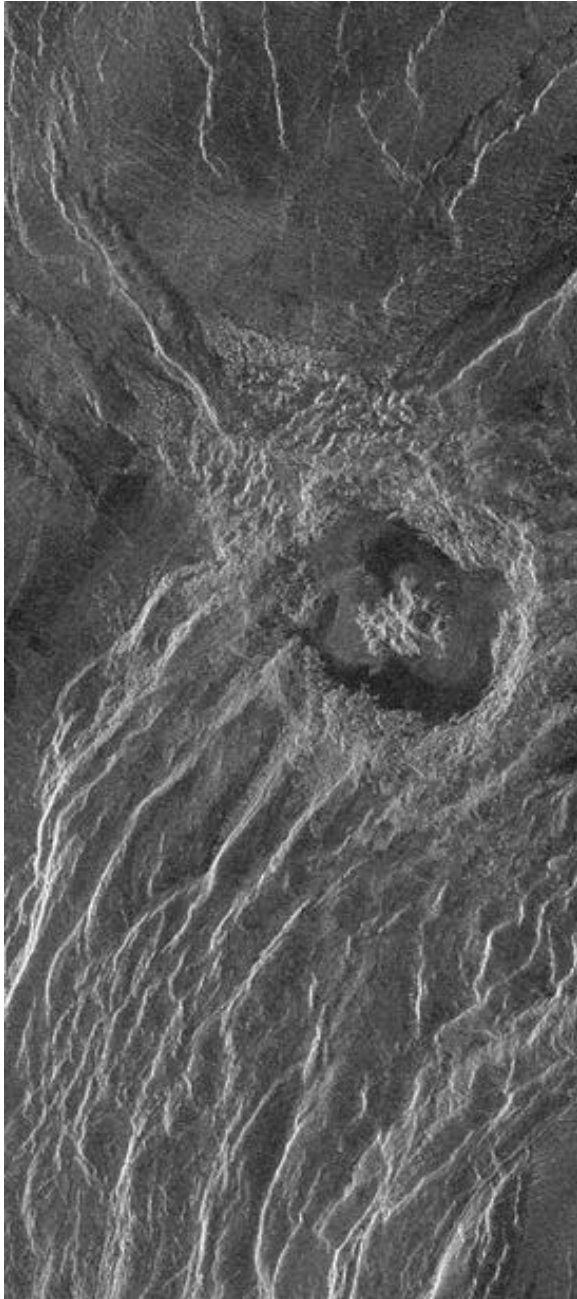
*Maria Goeppert Mayer*

3ħω  
odd



**Physicist**

**MARIA GOEPPERT MAYER**



Crater Goeppert Mayer  
on Venus



Maria Goeppert Mayer

1950: nuclear shell theory

1960: appointed professor  
at U. Calif. San Diego

1963: Nobel Prize



Maria Goeppert Mayer

1950: nuclear shell theory

1960: appointed professor  
at U. Calif. San Diego

1963: Nobel Prize

Headline in San Diego newspaper:  
“S.D. Mother Wins Nobel Prize”

Einstein  
and  
Bohr  
in 1930



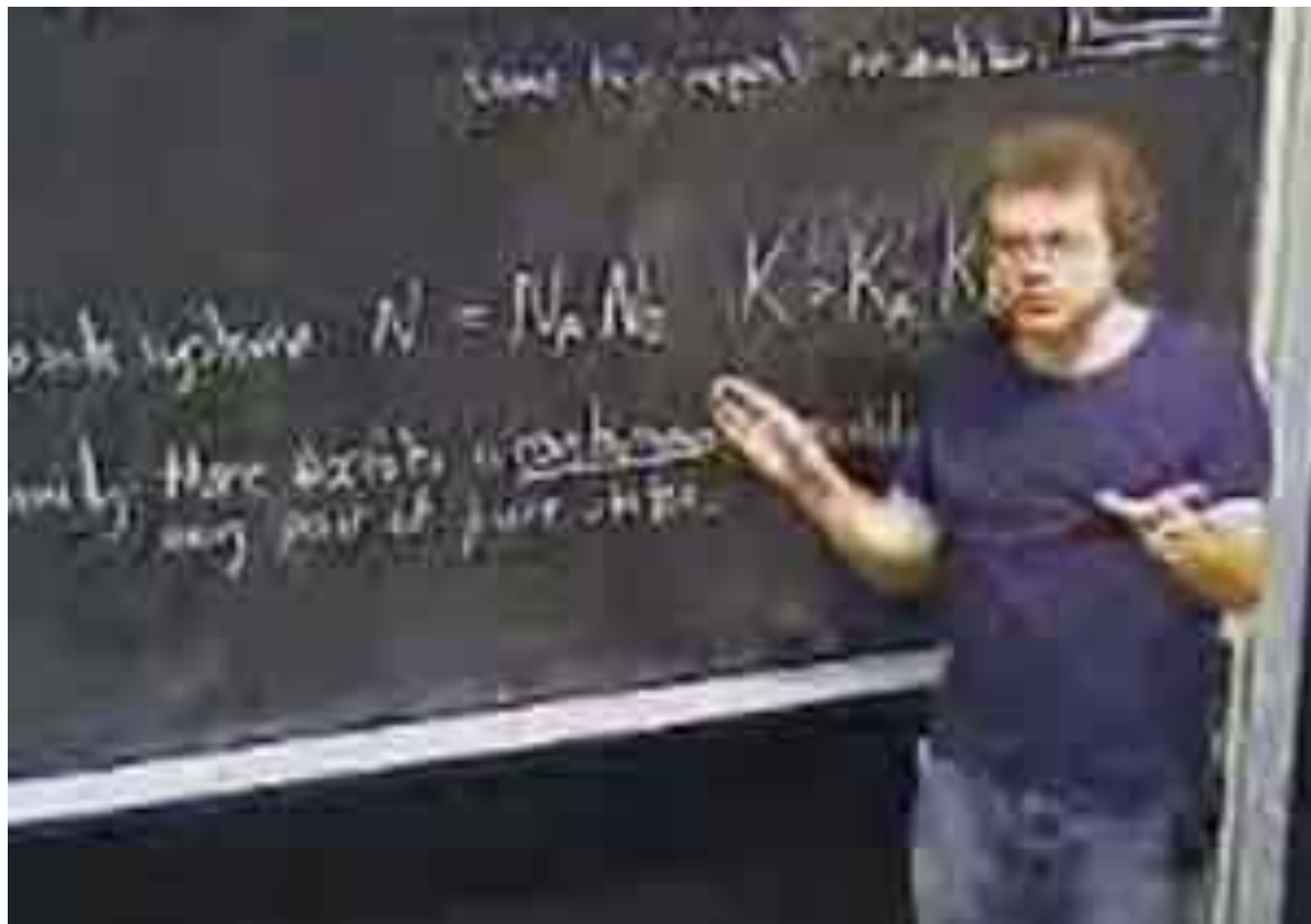


Einstein  
and  
Bohr  
in 1930

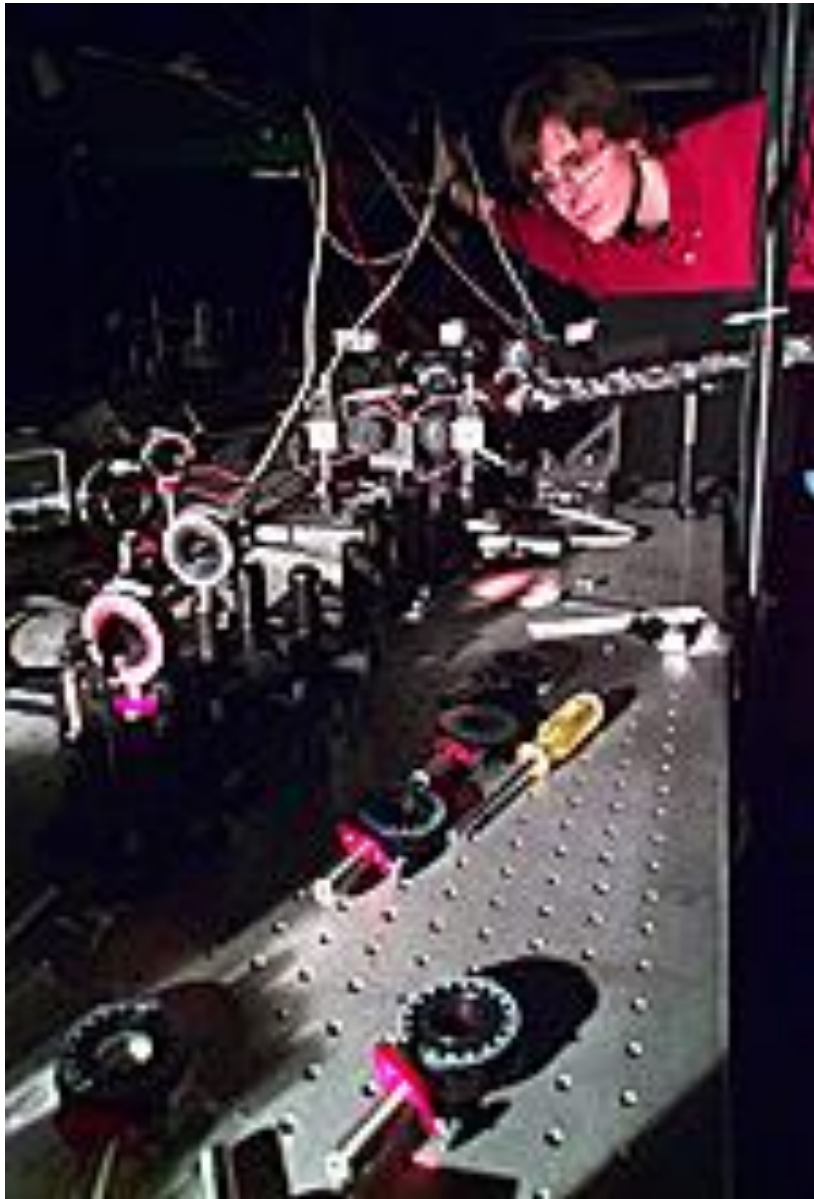




John Bell (age 36 in 1964)



Lucien Hardy



Paul Kwiat





Paul Kwiat

## Exceptional Bound States and Negative Entanglement Entropy

Ching Hua Lee \*

*Department of Physics, National University of Singapore, Singapore 117542, Singapore*

 (Received 9 December 2020; accepted 29 November 2021; published 7 January 2022)

This Letter introduces a new class of robust states known as exceptional bound (EB) states, which are distinct from the well-known topological and non-Hermitian skin boundary states. EB states occur in the presence of exceptional points, which are non-Hermitian critical points where eigenstates coalesce and fail to span the Hilbert space. This eigenspace defectiveness not only limits the accessibility of state information but also interplays with long-range order to give rise to singular propagators only possible in non-Hermitian settings. Their resultant EB eigenstates are characterized by robust anomalously large or negative occupation probabilities, unlike ordinary Fermi sea states whose probabilities lie between 0 and 1. EB states remain robust after a variety of quantum quenches and give rise to enigmatic negative entanglement entropy contributions. Through suitable perturbations, the coefficient of the logarithmic

## Exceptional Bound States and Negative Entanglement Entropy

Ching Hua Lee<sup></sup>\*

*Department of Physics, National University of Singapore, Singapore 117542, Singapore*

 (Received 9 December 2020; accepted 29 November 2021; published 7 January 2022)

This Letter introduces a new class of robust states known as exceptional bound (EB) states, which are distinct from the well-known topological and non-Hermitian skin boundary states. EB states occur in the presence of exceptional points, which are non-Hermitian critical points where eigenstates coalesce and fail to span the Hilbert space. This eigenspace defectiveness not only limits the accessibility of state information but also interplays with long-range order to give rise to singular propagators only possible in non-Hermitian settings. Their resultant EB eigenstates are characterized by robust anomalously large or negative occupation probabilities, unlike ordinary Fermi sea states whose probabilities lie between 0 and 1. EB states remain robust after a variety of quantum quenches and give rise to enigmatic negative entanglement entropy contributions. Through suitable perturbations, the coefficient of the logarithmic

## Exceptional Bound States and Negative Entanglement Entropy

Ching Hua Lee<sup></sup>\*

*Department of Physics, National University of Singapore, Singapore 117542, Singapore*

 (Received 9 December 2020; accepted 29 November 2021; published 7 January 2022)

This Letter introduces a new class of robust states known as exceptional bound (EB) states, which are distinct from the well-known topological and non-Hermitian skin boundary states. EB states occur in the presence of exceptional points, which are non-Hermitian critical points where eigenstates coalesce and fail to span the Hilbert space. This eigenspace defectiveness not only limits the accessibility of state information but also interplays with long-range order to give rise to singular propagators only possible in non-Hermitian settings. Their resultant EB eigenstates are characterized by robust anomalously large or negative occupation probabilities, unlike ordinary Fermi sea states whose probabilities lie between 0 and 1. EB states remain robust after a variety of quantum quenches and give rise to enigmatic negative entanglement entropy contributions. Through suitable perturbations, the coefficient of the logarithmic



# Exceptional Bound States and Negative Entanglement Entropy

Ching Hua Lee<sup>\*</sup>

Department of Physics, National University of Singapore, Singapore 117542, Singapore

(Received 9 December 2020; accepted 29 November 2021; published 7 January 2022)

This Letter introduces a new class of robust states known as exceptional bound (EB) states, which are distinct from the well-known topological and non-Hermitian skin boundary states. EB states occur in the presence of exceptional points, which are non-Hermitian critical points where eigenstates fail to span the Hilbert space. This eigenspace defectiveness not only limits the amount of information but also interplays with long-range order to give rise to singular propagation in non-Hermitian settings. Their resultant EB eigenstates are characterized by robustly negative occupation probabilities, unlike ordinary Fermi sea states whose probabilities are positive. EB states remain robust after a variety of quantum quenches and give rise to negative entanglement entropy contributions. Through suitable perturbations, the coefficient

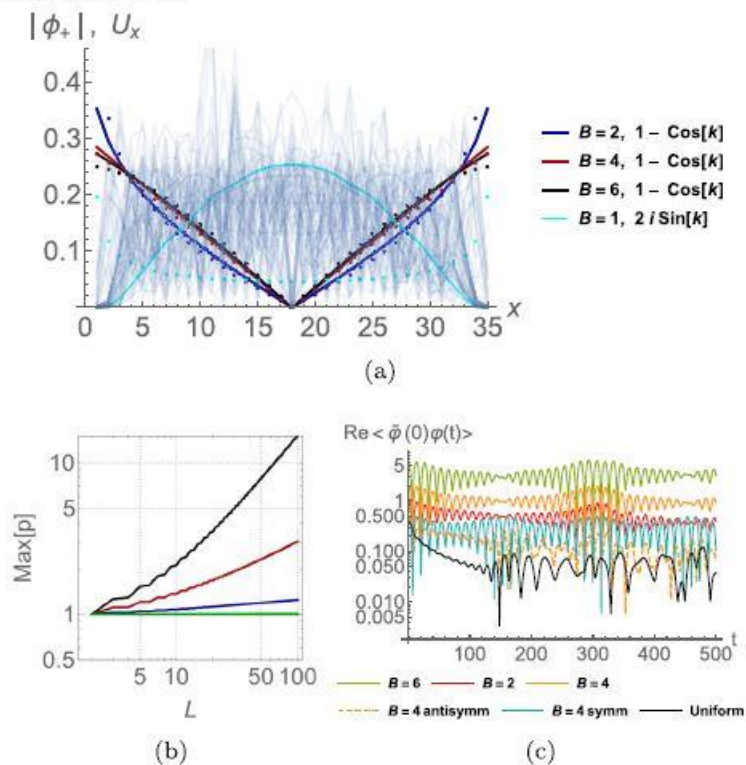


FIG. 2. (a) Numerical spatial lattice profiles of EB states  $|\phi_+\rangle$  (solid) vs the 2-site propagator  $U_x = \langle c_{x,+}^\dagger c_{0,-} \rangle$ . Close agreement is observed for sufficiently singular EPs with  $b(k)$  of  $B = 2, 4$  and 6th order, but not the  $B = 1$  case given by  $b(k) = 2i \sin k$ . Other non-EB states are superimposed as a gray background sea of states. (b) Anomalously large occupation probabilities  $p =$

# Exceptional Bound States and Negative Entanglement Entropy

Ching Hua Lee<sup>\*</sup>

Department of Physics, National University of Singapore, Singapore 117542, Singapore

(Received 9 December 2020; accepted 29 November 2021; published 7 January 2022)

This Letter introduces a new class of robust states known as exceptional bound (EB) states, which are initial skin boundary states. EB states occur in the vicinity of critical points where eigenstate degeneracy not only limits the number of states, but also give rise to singular propagators. These states are characterized by robustness against perturbations and give rise to anomalous scaling behaviors with system size  $L$  (up to constant multiplicative factors, which play no role in the emergence of EB states), as numerically verified in [89]:

$$b(k) = b_0[2(1 - \cos k)]^{B/2}, \quad (4a)$$

$$a(k) = b(-k) + a_0, \quad (4b)$$

which is realizable with lattice hoppings across at most  $B/2$  sites (see [89] for odd  $B$ ). Near the EP,  $\langle c_{x_1, -}^\dagger c_{x_2, +} \rangle \sim \sqrt{(b_0/a_0)} 2^{B/2} e^{-4(\Delta x)^2/B}$  is short-ranged, quadratically decaying with  $\Delta x = x_1 - x_2$ . However, due to the divergent denominator in  $U(k) \sim \sqrt{a_0/b(k)}$ , we have the following scaling behaviors with  $L$  (up to constant multiplicative factors, which play no role in the emergence of EB states), as numerically verified in [89]:

$$\langle c_{x_1, +}^\dagger c_{x_2, -} \rangle_{|B>4} \sim -\sqrt{\frac{a_0}{b_0}} \left(\frac{L}{\pi}\right)^{B/2-1} \times \left(2 - \frac{\pi^2 \Delta x^2}{L^2}\right), \quad (5)$$

which is long-ranged in  $\Delta x = x_1 - x_2$  and diverges as  $L^{B/2-1}$ . For the important cases of  $B = 2$  and  $B = 4$  [89],

$$\langle c_{x_1, +}^\dagger c_{x_2, -} \rangle_{|B=2} \sim -\sqrt{\frac{a_0}{b_0}} \left(\log \frac{L}{\pi \Delta x}\right), \quad (6)$$

$$\langle c_{x_1, +}^\dagger c_{x_2, -} \rangle_{|B=4} \sim -\sqrt{\frac{a_0}{b_0}} (L - 2\Delta x), \quad (7)$$

which diverges logarithmically and linearly with both  $L$  and  $x$ . Because of EP defectiveness, even a very small

initial skin boundary states. EB states occur in the vicinity of critical points where eigenstate degeneracy not only limits the number of states, but also give rise to singular propagators. These states are characterized by robustness against perturbations and give rise to anomalous scaling behaviors with system size  $L$  (up to constant multiplicative factors, which play no role in the emergence of EB states), as numerically verified in [89]:

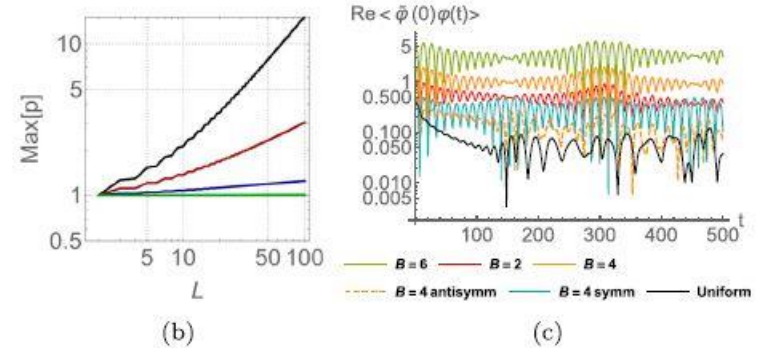
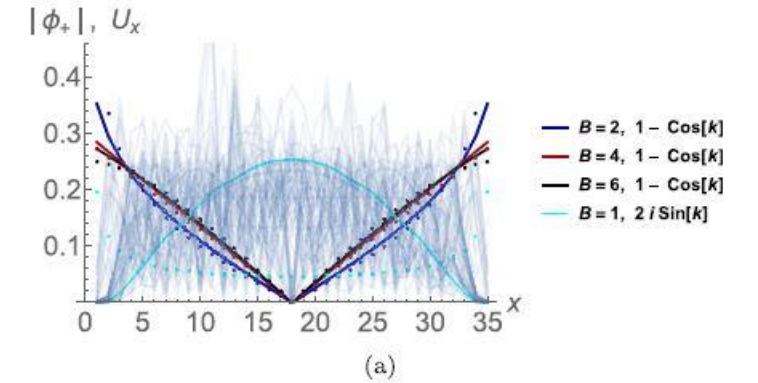


FIG. 2. (a) Numerical spatial lattice profiles of EB states  $\phi_+$  (solid) vs the 2-site propagator  $U_x = \langle c_{x_1, +}^\dagger c_{x_2, -} \rangle$ . Close agreement is observed for sufficiently singular EPs with  $b(k)$  of  $B = 2, 4$  and 6th order, but not the  $B = 1$  case given by  $b(k) = 2i \sin k$ . Other non-EB states are superimposed as a gray background sea of states. (b) Anomalous large occupation probabilities  $p = \langle \phi(0) \phi(t) \rangle$  vs system size  $L$ . (c) Anomalous large occupation probabilities  $p = \langle \phi(0) \phi(t) \rangle$  vs time  $t$ .



Werner Heisenberg (age 24 in 1925)





On this frigid winter morning, I hope you'll spend a moment of appreciation for our universe that is both weird and wonderful.