# The Price of Civil Society*

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**Abstract.** Most work in algorithmic game theory assumes that players ignore costs incurred by their fellow players. In this paper, we consider superimposing a social network over a game, where players are concerned with minimizing not only their own costs, but also the costs of their neighbors in the network. We aim to understand how properties of the underlying game are affected by this alteration to the standard model. The new social game has its own equilibria, and the *price of civil society* denotes the ratio of the social cost of the worst such equilibrium relative to the worst Nash equilibrium under standard selfish play. We initiate the study of the price of civil society in the context of a simple class of games. Counterintuitively, we show that when players become less selfish (optimizing over both themselves and their friends), the resulting outcomes may be worse than they would have been in the base game. We give tight bounds on this phenomenon in a simple class of load-balancing games, over arbitrary social networks, and present some extensions.

## 1 Introduction

The world of traditional game-theoretic analysis is a cold one. Each individual cares only about himself; he pays no heed to his neighbor's happiness. He makes strategic decisions with exclusive regard for his own direct preferences about the state of the world. He is not a good friend. Over the past decade, research on the *price of anarchy*—beginning with the seminal work of Koutsoupias and Papadimitriou [11], and progressing through landmark results like Roughgarden and Tardos's work on selfish routing [17], among many others—has flourished in the algorithmic game theory community. The price of anarchy measures the cost of this cold world: relative to a centrally planned optimum, how much worse are the outcomes that arise from purely selfish decision-makers?

But this picture of human decision-makers is unrealistically bleak. Humans maintain long-term dyadic relationships with nonrelatives; in other words, we have *friends*. The presence of this "friend" relationship is unusual among species, and it is a long-standing matter of research in the social sciences to explain its

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origins. (See Silk [18], for example.) Our daily lives are influenced deeply by what is called *civil society* by political scientists: that is, everything about society that is not “the state” or “the market.” Civil society is the church, the book club, the Association for Computing Machinery, the knitting circle. It is clear that our bonds with others affect our decisions; our relationships formed through civil society change our preferences relative to the solipsistic baseline. And, generally speaking, this effect is positive. (See Putnam’s *Bowling Alone* [15], for example.)

Our goal in this paper is to understand the way in which superimposing a social network on a strategic situation affects the quality of the resulting equilibria. To this end, we consider augmenting a “base game” with a social network. Each individual cares about both her own happiness and that of her neighbors in the social network. When the social network has no edges, we have “the market” and the classic Nash equilibrium. When the social network is the complete graph, we have “the state,” or at least individual agents myopically striving to optimize the welfare of society as a whole. Our interest lies in exploring the middle ground between selfishness and altruism. Specifically, we wish to analyze the following question: *How much better are the equilibria when, rather than acting in a purely selfish manner, players act on behalf of their friends as well as themselves?* That is, what benefits accrue from the presence of civil society?

Perhaps surprisingly, we observe that, by becoming *more* socially concerned, players can often end up at worse outcomes than standard Nash equilibria. This phenomenon has been observed in multiple related settings (e.g. [4, 9]), but our observation adds to the collection of such examples, suggesting that this effect is ubiquitous rather than pathological. Thus we are forced to address a different question: *How much* worse *are the equilibria when, rather than acting in a purely selfish manner, players act on behalf of their friends as well as themselves?*

We take a particular class of load-balancing game as our base games, and we examine the effect of varying the social network structure. We present tight constant bounds on the degradation of equilibrium quality in these games. Thus, in these games, we show that caring about friends can make the world worse, but only to a limited degree.

*The price of civil society.* Let $\Gamma$ denote an $n$-player game, and let $G$ denote an undirected graph on the players, where an edge reflects friendship between its endpoints. The *social game* $\Gamma_G$ has the same $n$ players and the same actual cost functions, but each player seeks to optimize the sum of her own cost *and* the costs of her neighbors in $G$. We are interested in the relative cost of the worst Nash equilibrium in the social game $\Gamma_G$ compared to the cost of the worst Nash equilibrium in the base game $\Gamma$. We call this ratio the *price of civil society.*

The complexity of this model lies in the structure of the social network. To isolate the effects of this structure, we limit our attention to *load-balancing games with identical linear machines.* Even in this simplistic setting, our model exhibits interesting and nontrivial behavior. Our main result is that, while the price of civil society can exceed 1, it does not exceed 5/4, and furthermore this bound is tight. The extremal example is a small game, and so we may wonder if this deterioration is merely a symptom of the discrete nature of atomic load-balancing.

We prove that it is not. As the number of players tends to infinity, and the game converges to its nonatomic version, the price of civil society becomes 9/8: smaller than in the atomic case, but still greater than 1. We also consider generalizations of this game and restrictions to particular classes of social networks.

*Related work.* Chen and Kempe [4] consider a routing game in which players optimize a linear combination of their own latency and the average latency of *all* players. The authors study the change in equilibria as a function of the weight given to the "altruistic" term, allowing for interpolation between totally selfish and totally altruistic behavior. Our model similarly interpolates between these extremes, but our "middle ground" reflects a player who cares about a select group of other players, rather than caring to a limited degree about all players.

Meier, Oswald, Schmid, and Wattenhofer [13] study a virus inoculation game in which players care about their neighbors' costs in addition to their own. Unlike our model, however, their game is intrinsically linked to the social network: changing the graph also changes the game. The primary goal of the present work is to study varied network topologies for a *fixed* game, thereby allowing us to isolate the impact of social structure on competitive play.

Ashlagi, Krysta, and Tennenholtz [1] introduce "social context games" which consider players who optimize general functions of their friends' utilities (rather than just the sum). Their work focuses on the existence of equilibria for a variety of functions, whereas we are concerned with the quality of these outcomes.

Hayrapetyan, Tardos, and Wexler [9] consider games in which players form coalitions, each modeled as a single player. Their model is closely related to the special case of our game in which the social network is a collection of cliques.

The price of anarchy was introduced by Koutsoupias and Papadimitriou [11] in the context of load-balancing games. Subsequent work has explored numerous variations of these games, including restricted classes of latency functions; symmetric or asymmetric access; unit or weighted jobs; mixed or pure equilibria; worst or best Nash equilibria; and sum or makespan objective functions. See, for example, [2, 3, 5–8, 10, 12, 19] and the references therein.

## 2 Model and Notation

For a game with player set $N$, a social network is given by an undirected graph with vertices $N$ and edges representing (symmetric) friendships between players. The *perceived cost* to player $i$ under a strategy profile $\mathbf{s}$ is the cost $i$ incurs plus the total costs incurred by all of $i$'s neighbors. The social cost $\textsc{sc}(\mathbf{s})$ of profile $\mathbf{s}$ is the sum of the actual (not perceived) costs experienced by each player.

We define OPT as a socially optimal strategy profile, that is, one with minimum social cost. An outcome is a pure Nash equilibrium (NE) when no individual player can decrease her actual cost by unilaterally switching to a different strategy; it is a *civil society Nash equilibrium* (CSNE) when no player can decrease her perceived cost by switching. Let WNE and WCSNE, respectively, denote the worst among the NE and the worst among the CSNE, measured by social cost.

The *price of anarchy* (POA) measures the extent to which the cost of any Nash equilibrium can exceed that of the optimal solution: it is SC(WNE)/SC(OPT). The *price of civil society* (POCS) measures the extent to which the cost of a civil society Nash equilibrium can exceed that of the worst pure Nash equilibrium. That is, the POCS is SC(WCSNE)/SC(WNE). Note that we are comparing worst social-game equilibria to worst base-game equilibria; it could also be interesting to consider *best* equilibria in one or both games.

In this paper, we consider a simple base game in which each player has access to a common set $M$ of resources. Each player selects a single resource, and incurs a cost equal to the total number of players who pick that resource. Given an outcome $\mathbf{s}$, the *load* $\ell_j = \ell_j(\mathbf{s})$ on a resource $j$ is the number of players who choose $j$. The social cost, given by $\sum_j \ell_j^2$, is completely determined by the load vector $L = (\ell_1, \ldots, \ell_m)$. This game is equivalent to load balancing with identical linear latencies. Such games are known to have pure Nash equilibria [14, 16]. Furthermore, it can be shown that for any social network $G$, these games also have civil society equilibria. In particular, a potential function for this game is $\Phi(\mathbf{s}) = \sum_{j \in M} \left[\binom{\ell_j+1}{2} + e_j\right]$, where $e_j$ is the number of edges with both endpoints choosing $j$ under $\mathbf{s}$: one can check that if a player switches to a new resource, the improvement in her perceived cost is exactly the decrease in $\Phi$, and so repeated best-response moves must eventually terminate at a CSNE.

We also consider the nonatomic version of this game. Informally, we picture a continuum of infinitesimally small players. More formally, let POCS$(n, m)$ denote the maximum POCS over all $n$-player, $m$-resource atomic games, and then define the nonatomic POCS to be $\sup_m \left(\limsup_{n\to\infty} \text{POCS}(n, m)\right)$.

## 3 Main Results

Consider an $n$-player, $m$-resource instance of our game. By convexity, the socially optimal solution assigns players to resources as evenly as possible. Furthermore, the only pure Nash equilibria are of the same form. Therefore WNE = OPT and thus POA = 1. Likewise, in the nonatomic version of this game, all resources have exactly the same load in both the optimal solution and the unique equilibrium. However, superimposing certain social networks on these games may cause the POCS to exceed 1; that is, some networks lead to social games with worse stable outcomes than without the network. We consider both the atomic and nonatomic settings, and prove that the POCS in these settings is 5/4 and 9/8, respectively.

We begin with some useful lemmas. Let $\delta$ be the size of a player in a game; that is, let $\delta = 1$ for atomic games and let $\delta \to 0$ for nonatomic games.

**Lemma 1.** *For any* CSNE *and any two resources* $1$ *and* $2$, $\ell_1 \leq 2\ell_2 + \delta$.

*Proof.* Consider a player $i$ using resource 1 in the CSNE. Suppose that she is friends with all players using resource 2 and with no one using resource 1: this arrangement is the case in which she will be most averse to switching to resource 2. As it stands, her cost is $\ell_1$, each of her $\ell_2/\delta$ friends' costs are $\ell_2$, and so her total perceived cost is $\ell_1 + \ell_2^2/\delta$. If she were to switch to resource 2, her cost would be $\ell_2 + \delta$, each of her $\ell_2/\delta$ friends' costs would be $\ell_2 + \delta$, and so her total perceived cost would be $\ell_2 + \delta + \ell_2^2/\delta + \ell_2$. Because she does not want to switch in a CSNE, we have that $\ell_1 + \ell_2^2/\delta \leq \ell_2 + \delta + \ell_2^2/\delta + \ell_2$. Simplifying, we get the desired $\ell_1 \leq 2\ell_2 + \delta$. This inequality holds for any social network. □

Given load vector $L = (\ell_1, \cdots, \ell_m)$, define $\Phi_m(L) = (\sum_i \ell_i^2)/\big[(\sum_i \ell_i)^2/m\big]$. This quantity is the ratio of the social cost of $L$ to that of a "fractional" Nash equilibrium (all loads are identical, even if that amount is not an integer), and thus is an upper bound on the POCS of this instance.

**Lemma 2.** *For a given machine $i$ and given bounds $a$ and $b$ with $0 \leq a \leq b$, suppose that we maximize $\Phi_m(L)$ over all $\ell_i$ such that $a \leq \ell_i \leq b$. Then this maximum will be achieved at either $\ell_i = a$ or $\ell_i = b$.*

*Proof.* We note that $\Phi_m(L)$ is not generally a convex function of $\ell_i$. However, it is continuously differentiable for $\ell_i \geq 0$, the derivative is zero at the unique point $x_i = (\sum_{j \neq i} \ell_j^2) / (\sum_{j \neq i} \ell_j)$, and the derivative is negative for $0 \leq \ell_i < x_i$ and positive for $\ell_i > x_i$. Thus the maximum occurs at either $\ell_i = a$ or $\ell_i = b$. □

### 3.1 Atomic Games

We begin with an example illustrating a nontrivial price of civil society. Consider four players, two resources, and social network $\mathcal{K}_{1,3}$, the complete bipartite graph on 1 and 3 vertices. Assigning the three leaf players to resource 1 and the root player to resource 2 yields a CSNE of cost 10. All NE have a cost of 8; thus, the POCS of this instance is 5/4. We will show that this example is in fact the worst possible, and therefore that the POCS for atomic games is 5/4. We first prove this result for any CSNE in which the load on each resource is at least 2:

**Lemma 3.** *Let $L$ be the load vector for a* CSNE *with minimum load at least 2. Then $\Phi_m(L) \leq 49/40 < 5/4$.*

*Proof.* Combining Lemmas 1 and 2, $\Phi_m(L)$ is maximized when all resources have load either $\ell$ or $2\ell + 1$ for some $\ell \geq 2$. Let $p$ and $\gamma \cdot p$ denote the number of resources with loads $\ell$ and $2\ell + 1$, respectively. Then $\Phi_m(L)$ is maximized when $\gamma = \ell/(2\ell + 1)$ and $\ell = 2$, giving an upper bound of $49/40 < 5/4$, as desired. □

Unfortunately, if some resources have a load of 0 or 1 in a CSNE with load vector $L$, then $\Phi_m(L)$ may exceed 5/4: the POCS is still bounded by 5/4, but $\Phi_m(L)$ uses a fractional Nash equilibrium, and more careful analysis is needed.

**Theorem 1.** *The price of civil society of the atomic game is* 5/4*.*

*Proof.* We have shown that that the POCS can be as large as 5/4. We must finish showing that 5/4 is an upper bound. Let $L$ be the load vector of a WCSNE. By Lemma 3, it only remains to consider the case in which the minimum load of $L$ is either 0 or 1. If the minimum load in $L$ is 0, then, by Lemma 1, every resource has load either 0 or 1. Such a configuration is a NE, and thus the POCS is 1. If the minimum load in $L$ is 1, Lemma 1 implies that the max load is 3. Let $p$, $q$, and $r$ denote the number of resources with loads 1, 2, and 3, respectively. The social cost of this outcome is $p+4q+9r$; to determine the POCS of this instance, it remains to determine SC(WNE) and optimize over $p$, $q$, and $r$.

Note that a Nash equilibrium cannot have both load-1 and load-3 resources: players will move from a load-3 to a load-1 resource until either the load-1 or load-3 resources are exhausted. Thus a Nash equilibrium either has no load-1 resources (if $p \leq r$) or no load-3 resources (if $r \geq p$). Suppose that $p \leq r$, so at NE, there are $(q+2p)$ load-2 and $(r-p)$ load-3 resources. The social cost of this configuration is thus $4(q+2p)+9(r-p) = -p+4q+9r$, and so POCS $= (p+4q+9r)/(-p+4q+9r)$. This expression is maximized when $p = r$ and $q = 0$, and evaluates to 5/4. Similar analysis shows that if $r \leq p$, the same bound holds. □

### 3.2 Nonatomic Games

In the atomic case, the worst instance for POCS had only a few players. We now consider nonatomic games, with an infinite number of infinitesimally small players. We show that while the price of civil society generally decreases as the number of players grows, it does not improve below 9/8. Thus, while a portion of our atomic POCS = 5/4 example can be attributed to an integrality issue (which tends to zero as the number of players grows) the remainder is due purely to the presence of socially conscious agents (the effects of which persist even in the nonatomic case).

We start with a nonatomic example whose POCS is 9/8. Let there be three resources and $n \to \infty$ players, with social network a complete tripartite graph with parts of size $n/4$, $n/4$, and $n/2$. The unique Nash equilibrium places $n/3$ players on each resource, for a social cost of $3(n/3)^2 = n^2/3$. There is a CSNE that places the $n/2$-player part on resource 1 and the other parts on resources 2 and 3, for a social cost of $3n^2/8$, yielding a POCS of 9/8.

**Theorem 2.** *The price of civil society of the nonatomic game is* 9/8.

*Proof.* In light of the previous example, we need only prove an upper bound. It suffices to show $\Phi_m(L) \leq 9/8$ for a WCSNE with load vector $L$. Lemmas 1 and 2 together imply that the maximum $\Phi_m(L)$ occurs when each $\ell_i$ is either $\ell$ or $2\ell$, for some $\ell > 0$. Let $p$ and $q$ denote the number of resources with loads $\ell$ and $2\ell$, respectively. Then $\Phi_m(L)$ is maximized at $q = p/2$, where the ratio is 9/8. □

## 4 Extensions and Future Work

The POCS of 5/4 for the discrete game was only achievable on a complete bipartite graph. These graphs have *no* triadic closure: friends of friends are never friends. But real social networks exhibit a high degree of triadic closure [20]. What happens if we force the graph to be more like a friendship-based social network?

We first look at a graph with *complete* triadic closure (i.e., all connected components are cliques). Here, loads in a CSNE must be as balanced as possible (or else at least one clique would have more people using some overloaded resource than some underloaded one). Therefore every CSNE is a NE, and the POCS is 1: there is no degradation because of the social network.

What happens with graphs between these extremes? What "intermediate" social structures should we examine? Here is one option. Precisely define the triadic closure of a graph to be the probability $\Delta$ that a path of length two, chosen uniformly at random, is part of a triangle in the graph [20]. What happens as we vary $\Delta$ in our social network? Unfortunately, graphs with $\Delta$ approaching 1 can have the worst possible POCS of 5/4 (using many copies of the original worst-case example). However, for a fixed number of resources, restricting the allowable triadic closure can provide improved bounds on the worst possible POCS. We have seen that with no constraints on $\Delta$, the POCS can be as large as 5/4, while if $\Delta$ is 1, the POCS is 1. Many intermediate results are possible: e.g., with two resources, we can show that if $\Delta > 7/11$, the POCS is at most 10/9.

Another way to generalize our results is to change our base game beyond a load-balancing game with unit-weight jobs and machines with identical linear latencies. We have proven several such results that we simply state here without proof. (The proof methods are substantially similar to those in Section 3.)

First we consider *related machines*, where machine $j$ has latency function $a_j\ell$ under load $\ell$. For two machines with atomic players, we can show that the price of civil society is $14/11 \approx 1.27$, a mild worsening over our earlier bound of $5/4 = 1.25$ for identical machines. The extremal example has two jobs on a machine of latency $3\ell$ and one job on a machine of latency $2\ell$. For nonatomic players, the POCS remains $9/8 = 1.125$, the same as for identical machines. Unsurprisingly, for broader classes of latency functions, the POCS increases substantially: for general convex and increasing latencies, the POCS reaches $n$, the number of players.

Another generalization is *weighted* load balancing, where jobs may have different sizes. For two identical linear machines, the POCS increases to $34/25 = 1.36$. The extremal example has one machine with two jobs of weight 4 and one machine with two jobs of weight 1. For more than two identical linear machines, we can prove that the POCS lies in a narrow range slightly greater than 1.5.

More generally, similar analysis can be applied to arbitrary base games. We suspect that for some classes of games, social networks may cause substantially greater degradation in the resulting equilibria, while in others, social structure may only improve outcomes. In what games are social networks most harmful?

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