Quiz 2 Key

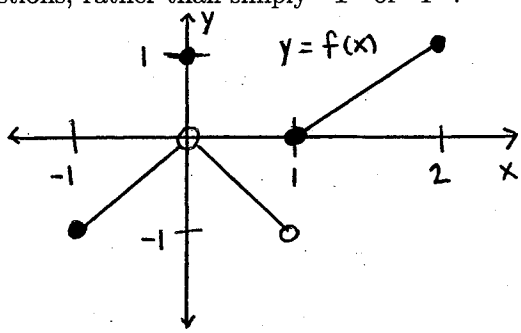
Math 221, Lecture 5, Sections 398/403 - Fall 2009

Name: _____ Section: _____ Date: _____ Score: ____/20

You must show all of your work to get any credit. Partial credit may be given where applicable.

True/False (1 pt. each)

From the graph below, determine whether the following statements are true or false. Answer "True" or "False" to the following questions, rather than simply "T" or "F".



1. True $\lim_{x\to 0} f(x)$ exists.
2. False $\lim_{x\to 0} f(x) = 1$.
3. True $\lim_{x\to 0} f(x) = 0$.
4. False $\lim_{x\to 1} f(x) = 1$.
5. False $\lim_{x\to 1} f(x) = 0$.

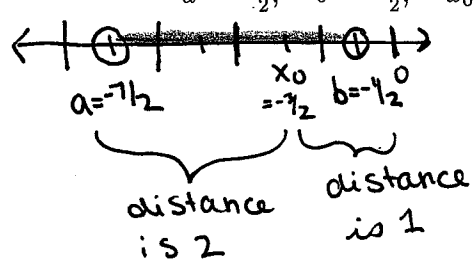
   Limit does not exist.
6. True $\lim_{x\to x_0} f(x)$ exists at every point x_0 in $(-1, 1)$.

Problem Section

Solve each of the following problems. Show all of your work. (i.e. u = 1-5x, etc.)

1. (4 points) Sketch the interval (a, b) on the x-axis with the point x_0 inside. Then find a value of $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \implies a < x < b$.

$$a = -\tfrac{7}{2}, \quad b = -\tfrac{1}{2}, \quad x_0 = -\tfrac{3}{2}$$



$\delta = \min(1, 2) = 1.$

You could also have picked $\delta \leq 1$.

2. (10 points) Find an open interval about x_0 on which the inequality $|f(x) - L| < \epsilon$ holds. Then, give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$, the inequality $|f(x) - L| < \epsilon$ holds.

NOTE: You aren't being asked to write a formal proof. Just find a δ that works.

$$f(x) = \sqrt{x+1}$$
$$L = 1$$
$$x_0 = 0$$

You can do this problem the same way you would do it if ε was given to you. [Say $\varepsilon = .5$ for example]

$$|f(x) - L| < \varepsilon$$
$$|\sqrt{x+1} - 1| < \varepsilon$$
$$-\varepsilon < \sqrt{x+1} - 1 < \varepsilon$$
$$1 - \varepsilon < \sqrt{x+1} < 1 + \varepsilon$$
$$(1-\varepsilon)^2 < x+1 < (1+\varepsilon)^2$$
$$(1-\varepsilon)^2 - 1 < x < (1+\varepsilon)^2 - 1$$

This step is really only allowed when each term is greater than or equal to zero. So, we'll assume $\varepsilon \leq 1$.

Note that in this problem $x_0 = 0$ and so we're looking for

$$-\delta < x < \delta.$$

Thus, $\delta = \min\left(|(1-\varepsilon)^2 - 1|, |(1+\varepsilon)^2 - 1|\right)$.