Quiz 3 Key

Math 221, Lecture 5, Sections 398/403 - Fall 2009

Name: Key Section: _____ Date: _____ Score: ____/20

You must show all of your work to get any credit. Partial credit may be given where applicable.

Problem Section

Solve each of the following problems. Show all of your work. (i.e. u = 1-5x, etc.)

1. (5 points each) Find the derivative of each of the following functions. You do not need to simplify your answer.

   (a) $f(x) = \frac{cosx+\sqrt{x}}{x}$

$$f'(x) = \frac{x(\cos x+\sqrt{x})' - (x)'(\cos x+\sqrt{x})}{x^2} \quad \text{Quotient Rule}$$

$$= \frac{x(-\sin x + \tfrac{1}{2}x^{-1/2}) - 1\cdot(\cos x+\sqrt{x})}{x^2}$$

$$= \frac{-x\sin x + \tfrac{1}{2}x^{1/2} - \cos x - \sqrt{x}}{x^2}$$

$$= \frac{-x\sin x - \cos x - \sqrt{x}}{}$$

   (b) $f(x) = \sqrt{x+\sqrt{x}}$

$$f(x) = (x + x^{1/2})^{1/2}$$

$$f'(x) = \tfrac{1}{2}(x+x^{1/2})^{-1/2}(1+\tfrac{1}{2}x^{-1/2})$$

   (c) $f(x) = cos^2(cosx) + sin^2(cosx)$

$f(x) = 1$ Recall $\cos^2 x + \sin^2 x = 1$.

$f'(x) = 0$.

You could do the chain rule here.

$$f(x) = (\cos(\cos x))^2 + (\sin(\cos x))^2$$

$$f'(x) = 2\cos(\cos x)(-\sin(\cos x))(-\sin x) + 2\sin(\cos x)(\cos(\cos x))(-\sin x)$$

$$= 2\cos(\cos x)\sin(\cos x)(\sin x) - 2\cos(\cos x)\sin(\cos x)(\sin x)$$

$$= 0.$$

2. (5 points) Oh no! You are taking the second midterm and you realize that you've completely forgotten what the derivative of $tanx$ is! Find the derivative of $f(x) = tanx$. Just stating what it is will not be sufficient. You have to "discover" it for yourself through some method. You should simplify $f'(x)$ into a form that we would recognize.

Recall that $\tan x = \frac{\sin x}{\cos x}$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x(\sin x)' - \sin x(\cos x)'}{\cos^2 x} \quad \text{(Quotient Rule)}$$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x.$$

Thus, the derivative of $\tan x$ is $\sec^2 x$.