Quiz 5 Key

Math 221, Lecture 5, Sections 398/403 - Fall 2009

Partial credit may be given where applicable.

True/False (1 pt. each)

Determine whether the following statements are true or false. Answer "True" or "False" to the following questions, rather than simply "T" or "F".

1. <u>False</u> Every function has an inverse.
   Explanation: Only 1-1 functions have inverses.

2. <u>False</u> $(f^{-1})'(b) = \dfrac{1}{f^{-1}(f'(b))}$.
   Explanation: The formula is incorrect. It should be $(f^{-1})'(b) = \dfrac{1}{f'(f^{-1}(b))}$.

3. <u>True</u> $ln(x^r) = rlnx$.

4. <u>False</u> $\frac{d}{dx}sec^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$.
   Explanation: That is the formula for $\dfrac{d}{dx}tan^{-1}u$.
   The correct formula is $\dfrac{d}{dx}sec^{-1}(u) = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

5. <u>False</u> $ln(x+a) = lnx + lna$.
   Explanation: The correct formula is $ln(ax) = lna + lnx$.

6. <u>True</u> $\dfrac{d}{dx}sin^{-1}(u) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$, $|u| < 1$.

Problem Section

Solve each of the following problems. Show all of your work. (i.e. u = 1-5x, etc.)

1. (4 points) Let $f(x) = x^3 + 2x - 1$. Find $\frac{df^{-1}}{dx}$ at $x = 11 = f(2)$.

   Solution:

   Note that $f'(x) = 3x^2 + 2$.

$$\begin{aligned}\frac{df^{-1}}{dx}(11) &= \frac{1}{f'(f^{-1}(11))}\\ &= \frac{1}{f'(2)}\\ &= \frac{1}{3(2^2)+2}\\ &= \frac{1}{14}\end{aligned}$$

2. (4 points) Find the inverse of $y = x^3 - 2$.

Solution:

Solve for x:

$$\begin{aligned} y &= x^3 - 2 \\ y + 2 &= x^3 \\ (y+2)^{\frac{1}{3}} &= x \end{aligned}$$

Switch x and y:

$$y = (x+2)^{\frac{1}{3}}.$$

Thus, $y = (x+2)^{\frac{1}{3}}$ is the inverse of $y = x^3 - 2$.

3. (6 points) Calculate the derivative of $f(x)$ by a method of your choice. (Choose wisely!) You don't need to simplify your answer.

$$f(x) = ln\left(\sqrt{\frac{x^{\frac{1}{7}}(x-2)^3(tan^{-1}(x))}{x^2+1}}\right)$$

Solution:

There's an easy and a hard way to do this problem. You can try to take the derivative without simplifying at all, and this is a legitimate way to do the problem. However, it will be much harder on you than if you'd thought to simplify first. The solution given here will be the easiest one and the one where you are the least likely to make a ton of algebra mistakes.

We start by using some properties of the natural log function.

$$\begin{aligned} f(x) &= ln\left(\sqrt{\frac{x^{\frac{1}{7}}(x-2)^3(tan^{-1}(x))}{x^2+1}}\right) \\ &= ln\left(\left(\frac{x^{\frac{1}{7}}(x-2)^3(tan^{-1}(x))}{x^2+1}\right)^{\frac{1}{2}}\right) \\ &= \frac{1}{2}ln\left(\frac{x^{\frac{1}{7}}(x-2)^3(tan^{-1}(x))}{x^2+1}\right) \\ &= \frac{1}{2}\left[ln\left(x^{\frac{1}{7}}(x-2)^3(tan^{-1}(x))\right) - ln\left(x^2+1\right)\right] \\ &= \frac{1}{2}\left[ln\left(x^{\frac{1}{7}}\right) + ln\left((x-2)^3\right) + ln\left(tan^{-1}(x)\right) - ln\left(x^2+1\right)\right] \\ &= \frac{1}{2}\left[\frac{1}{7}lnx + 3ln(x-2) + ln\left(tan^{-1}(x)\right) - ln\left(x^2+1\right)\right] \end{aligned}$$

Now that our equation is as simplified as possible, we will take the derivative.

$$
\begin{aligned}
f'(x) &= \left(\frac{1}{2}\left[\frac{1}{7}lnx + 3ln(x-2) + ln\left(tan^{-1}(x)\right) - ln\left(x^2+1\right)\right]\right)' \\
&= \frac{1}{2}\left[\frac{1}{7}\frac{1}{x} + 3\frac{1}{x-2} + \frac{1}{1+x^2} - \frac{1}{x^2+1}\cdot(2x)\right] \\
&= \frac{1}{2}\left[\frac{1}{7}\frac{1}{x} + 3\frac{1}{x-2} + \frac{1}{1+x^2} - \frac{2x}{x^2+1}\right] \\
&= \frac{1}{2}\left[\frac{1}{7x} + \frac{3}{x-2} + \frac{1-2x}{1+x^2}\right]
\end{aligned}
$$