Quiz 6 Key

Math 221, Lecture 2 - Fall 2010

Name: Answer Key Section: _____ Date: _____ Score: ____/10

There are 10 points possible. Partial credit may be given where applicable.
Solve each of the following problems. Show all of your work. (i.e. u = 1-5x, etc.) If you run out of room, you may use additional sheets of paper. **Be aware of problems on the back of the page.**

1. (5 points) Find the absolute maximum and minimum values of

$$g(x) = -\sqrt{5 - x^2}$$

on the interval $-\sqrt{5} \leq x \leq 0$ and say where they are assumed.

$$g'(x) = -\frac{1}{2}(5-x^2)^{-\frac{1}{2}}(5-x^2)'$$

$$= -\frac{1}{2}(5-x^2)^{-1/2}(-2x)$$

$$= \frac{x}{\sqrt{5-x^2}}$$

Critical pts: $x=0, \sqrt{5}, -\sqrt{5}$.

Only $x=0$ and $x=-\sqrt{5}$ are in our interval, so we discard $x=\sqrt{5}$.

$g(0) = -\sqrt{5-0^2} = -\sqrt{5}$

$g(-\sqrt{5}) = -\sqrt{5-(-\sqrt{5})^2} = -\sqrt{5-5} = 0.$

Absolute max @ $(-\sqrt{5}, 0)$

Absolute min @ $(0, -\sqrt{5})$

2. (5 points) For what values of a, m and b does the function

$$f(x) = \begin{cases} 3 & \text{if } x = 0 \\ -x^2 + 3x + a & \text{if } 0 < x < 1 \\ mx + b & \text{if } 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

WANT $f(x)$ continuous on $[0,2]$
$f(x)$ differentiable on $(0,2)$

continuous

$$f(0) = \lim_{x \to 0^+} f(x)$$

$$3 = \lim_{x \to 0^+} (-x^2 + 3x + a)$$

$$3 = -0 + 0 + a$$

$$\boxed{3 = a}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$$

$$\lim_{x \to 1^+} (mx + b) = \lim_{x \to 1^-} (-x^2 + 3x + 3)$$

$$m + b = -1 + 3(1) + 3$$

$$\boxed{m + b = 5}$$

differentiable

$$f'(x) = \begin{cases} -2x + 3 & 0 < x < 1 \\ ? & x = 1 \\ m & 1 < x < 2 \end{cases}$$

$$\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^-} f'(x)$$

$$\lim_{x \to 1^+} (m) = \lim_{x \to 1^-} (-2x + 3)$$

$$m = -2(1) + 3 \Rightarrow \boxed{m = 1}$$

$$\boxed{\begin{aligned} a &= 3 \\ m &= 1 \\ b &= 4 \end{aligned}}$$