Exercise 10 solution:

(a) Let k be an integer. We have

$$\lim_{n\to\infty} \frac{F^n(x+k)-(x+k)}{n} = \lim_{n\to\infty} \frac{F^n(x)+k-x-k}{n} = \lim_{n\to\infty} \frac{F^n(x)-x}{n}.$$

Thus, to show $\rho(f)$ is the same for $x_1 \neq x_2$, it suffices to consider $x_1, x_2 \in [0,1)$, so that $x_2 - 1 \leq x_1 \leq x_2 + 1$.

(b) Recalling that F is strictly increasing on $\mathbf{R}$, we have

$$\begin{aligned}\lim_{n\to\infty} \frac{F^n(x_2)-x_2}{n} &= \lim_{n\to\infty} \frac{F^n(x_2)-1-x_2-1}{n} = \lim_{n\to\infty} \frac{F^n(x_2-1)-(x_2+1)}{n} \\ &\leq \lim_{n\to\infty} \frac{F^n(x_1)-x_1}{n} = \lim_{n\to\infty} \frac{F^n(x_1)-1-x_1-1}{n} \\ &= \lim_{n\to\infty} \frac{F^n(x_1-1)-(x_1+1)}{n} \leq \lim_{n\to\infty} \frac{F^n(x_2)-x_2}{n}.\end{aligned}$$

Hence $\rho(f)$ is independent of x.