Exercise 18 solution: Suppose H is discontinuous at $y_0 \in \mathbf{R}$. Since H is increasing, there exist sequences $\{u_i\} \to y_0$ and $\{v_i\} \to y_0$ as $i \to \infty$ satisfying $u_i < u_{i+1},\ v_{i+1} < v_i$ for all i, and

$$a = \lim_{i \to \infty} H(u_i) < \lim_{i \to \infty} H(v_i) = b.$$

Again using the fact H is increasing, we then have $H(\mathbf{R}) \cap (a, b) = \emptyset$. However, $H(eo(x_0)) = \{\, i\alpha + j \ : \ i, j \in \mathbf{Z} \,\} = eo(0, R_\alpha)$ is dense in $\mathbf{R}$ by Exercise 8(b). This contradicts $H(\mathbf{R}) \cap (a, b) = \emptyset$; hence there is no such point y_0, and H is continuous on $\mathbf{R}$.