Exercise 22 solution: Let $0 < \epsilon \leq 1$. Let $x \in \mathbf{R}$. $F_{\omega,\epsilon}(x+1) = x + 1 + \omega + \frac{\epsilon}{2\pi}\sin(2\pi(x+1)) = F_{\omega,\epsilon}(x) + 1$. Hence $F_{\omega,\epsilon}$ is the lift of a circle map $f_{\omega,\epsilon}$.

Since $\epsilon \leq 1$, $F'_{\omega,\epsilon}(x) = 1 + \epsilon\cos(2\pi x) = 0$ if and only if $\epsilon = 1$ and $x = 0.5 + k$, $k \in \mathbf{Z}$. If $\epsilon < 1$ then $F'_{\omega,\epsilon}(x) > 0$ for all x and $F_{\omega,\epsilon}$ is a diffeomorphism. If $\epsilon = 1$, the reader may check that $x = 0.5 + k$ is an inflection point of $F_{\omega,1}$. If $x \neq 0.5 + k$ then $F'_{\omega,1}(x) > 0$, so that $F_{\omega,1}$ is strictly increasing and thus a homeomorphism. We have $F_{\omega,\epsilon}$ is the lift of an orientation preserving circle homeomorphism $f_{\omega,\epsilon}$ for $0 < \epsilon \leq 1$.