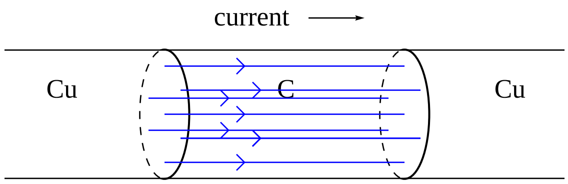
Electric fields and charges in resistors

(a.)

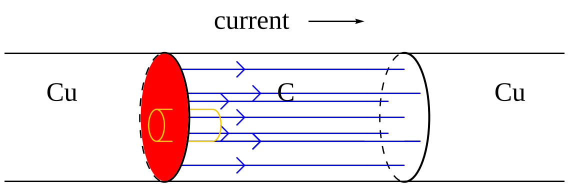


In Cu to left $\vec{E} = \rho_{\rm Cu}\vec{J} = 0$ because $\rho_{\rm Cu} = 0$.

In C at center $\vec{E} = \rho_{\rm C}\vec{J}$ is uniform and points rightward because $\vec{J}$ is uniform and points rightward.

In Cu to right $\vec{E} = \rho_{\rm Cu}\vec{J} = 0$ because $\rho_{\rm Cu} = 0$.

(b.) What is the source of this $\vec{E}$? There must be some charge at the Cu–C boundary! The figure below shows this charge on the left-hand boundary, and shows a Gaussian cylinder straddling that boundary.



The Gaussian cylinder has cap area A and
charge enclosed $= \sigma A$
flux through left cap $= 0$ (because $\vec{E} = 0$)
flux through sides $= 0$ (because $\vec{E}$ is parallel to sides)
flux through right cap $= |\vec{E}|A = \rho_{\rm C} JA$

And by Gauss's law,

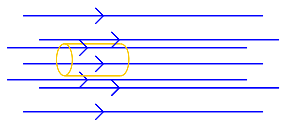
$$\begin{aligned} \Phi &= \frac{Q_{\rm inside}}{\epsilon_0} \\ \rho_{\rm C} JA &= \frac{\sigma A}{\epsilon_0} \\ \sigma &= \epsilon_0 \rho_{\rm C} J \end{aligned}$$

At the right-hand C–Cu boundary, exactly the same reasoning applies except that the flux is all through the left cap, and $\Phi = -|\vec{E}|A$, so

$$\sigma = -\epsilon_0 \rho_{\rm C} J.$$

As you would suspect, the charge buildup is bigger for bigger currents, and bigger for bigger resistivities.

(c.) Situate the Gaussian cylinder completely within any region where the field is uniform (including uniformly zero):



For any such cylinder,

$$\begin{aligned} \text{flux through sides} &= 0 \\ \text{flux through left cap} &= -\text{flux through right cap} \end{aligned}$$

so the total flux vanishes and the charge enclosed must be zero. Since we can apply this trick to *any* cylinder, no matter how small, the charge density is everywhere zero.

Grading: Part (a), 3 points.
Part (b), charge density on left face, 3 points.
Part (b), charge density on right face, 1 point.
Part (c), 3 points.