Electric potential due to a quarter disk

Solution A (Invented by me.)

This problem breaks apart into three distinct pieces:

1. Get the concepts straight. From superposition,

$$\begin{aligned}\text{potential due to full disk} &= \text{potential due to north quadrant} + \text{potential due to east quadrant} + \\ &\quad \text{potential due to south quadrant} + \text{potential due to west quadrant.}\end{aligned}$$

But at any point immediately above the sharp tip of the pie slice, symmetry demands that

$$\begin{aligned}\text{potential due to north quadrant} &= \text{potential due to east quadrant} \\ &= \text{potential due to south quadrant} \\ &= \text{potential due to west quadrant.}\end{aligned}$$

So at any point immediately above the sharp tip of the pie slice,

$$\text{potential due to a quadrant} = \tfrac{1}{4} \times \text{potential due to full disk.}$$

2. Find the formula from the concepts. The formula for the potential due to a full disk is given by LSM as the last equation of example 7.15, "Potential Due to a Uniform Disk of Charge" (pages 305–306). We need one quarter of that or

$$\frac{1}{4}k_e\,2\pi\sigma\left(\sqrt{z^2+R^2}-z\right) = \frac{\pi}{2}k_e\sigma\left(\sqrt{z^2+R^2}-z\right). \tag{1}$$

3. Put numbers into the formula. Converting distances into meters and using three significant digits, this formula gives the answer

$$47.1\ \mu\text{V}.$$

Solution B (Invented by Megan Kyi and Solomon Chang, class of 2026.)

Follow the reasoning of LSM examples 7.14, "Potential Due to a Ring of Charge", and 7.15, "Potential Due to a Uniform Disk of Charge" (pages 305–306), but on page 305, don't integrate from 0 to 2π, instead integrate from 0 to $\pi/2$. The result will be $\frac{1}{4}$ of the last equation of example 7.15 on page 306, namely

$$\frac{1}{4}k_e\,2\pi\sigma\left(\sqrt{z^2+R^2}-z\right) = \frac{\pi}{2}k_e\sigma\left(\sqrt{z^2+R^2}-z\right). \tag{2}$$

You can put numbers into the formula (be sure to convert distances into meters and to use three significant digits) giving the answer

$$47.1\ \mu\text{V}.$$

Grading using my strategy: 3 points for the idea that you want $\frac{1}{4}$ the potential from the full disk. The reasoning can be as telegraphic as "By symmetry", but there must be some reasoning.

4 points for the equation (1).

1 point for the number.

1 point for the units.

1 point for three significant figures.

Grading using Megan/Solomon strategy: 3 points for the idea that the integral for the pie slice will be $\frac{1}{4}$ the integral for the full pie.

4 points for the equation (2).

1 point for the number.

1 point for the units.

1 point for three significant figures.