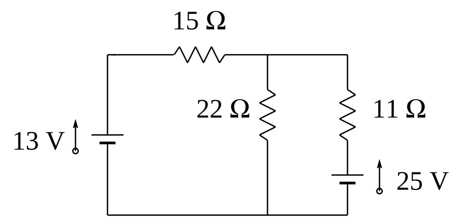
Network circuit

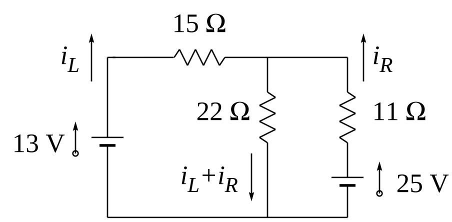
Problem: What current flows upward through the 13 V battery?



Model solution

Select strategy. My first thought is to solve this problem using the rules for combining resistors in series and in parallel. But no! The 11 Ω resistor is not in parallel with the 22 Ω resistor, because the 25 V battery is in the way: these two resistors do *not* experience the same voltage drop. And the 15 Ω resistor is not in series with the 22 Ω resistor, because of the branch to the 25 V battery: these two resistors do *not* carry the same current. So we have to go back to Kirchhoff's rules to solve the circuit.

Set up.



Call the current up through the left battery i_L (this is our desired quantity), the current up through the right battery i_R. Then (node rule) the current down through the 22 Ω resistor is $i_L + i_R$. ⟦Mnemonic names like i_L and i_R are more memorable than arbitrary names like i' and i''. I don't give a variable name to the combination $i_L + i_R$ to avoid a long list of variable names for things I don't want to find.⟧

Use physics to find the relevant equations. The loop rule applied to the inner left loop gives

$$13\text{ V} - i_L(15\ \Omega) - (i_L + i_R)(22\ \Omega) = 0$$

while the loop rule applied to the inner right loop gives

$$25\text{ V} - i_R(11\ \Omega) - (i_L + i_R)(22\ \Omega) = 0.$$

⟦There's no need to apply the loop rule to the outer loop — that will just give the difference of these two inner loop equations. And if you write down that third equation, the larger thicket of equations will make the solution harder to find.⟧

Use math to solve those equations. So we need to solve simultaneously

$$\begin{aligned} 13\text{ V} - i_L(37\ \Omega) - i_R(22\ \Omega) &= 0 \\ 25\text{ V} - i_L(22\ \Omega) - i_R(33\ \Omega) &= 0. \end{aligned}$$

We want to find i_L and eliminate i_R. To do this, multiply the top equation by 3, and the bottom equation by -2, then sum:

$$(3 \times 13 - 2 \times 25)\text{ V} - i_L(3 \times 37 - 2 \times 22)\ \Omega = 0$$

giving

$$i_L = \frac{(3 \times 13 - 2 \times 25)}{(3 \times 37 - 2 \times 22)}\text{ A} = \frac{-11}{67}\text{ A} = -0.16\text{ A}.$$

Express answer appropriately. Because all the data have two significant figures, the result is reported to two significant figures. And of course it contains the unit of "ampere".

Reflection: Note the negative sign. The 25 V battery is so hefty that it actually drives current *down* through the 13 V battery. ⟦This reflection is not required for the solution, but really it's the whole point of the problem. I assigned this problem to make you realize that while current usually flows in the direction that the battery drives it, it doesn't always.⟧

Grading: 2 points for starting off (e.g. diagram, variable names)
1 point for node rule (explicit or implicit)
2 points for left loop equation
2 points for right loop equation
2 points for numerical solution
1 point for two significant figures