

Parallel RC circuit

a. $\Delta V_R(t) = i_R(t)R$, so $i_R(t) = \frac{\mathcal{E}(t)}{R} = \frac{\mathcal{E}_m}{R} \sin(\omega t)$.

b. $C = \frac{q(t)}{\Delta V_C(t)}$, so $q(t) = C\mathcal{E}(t) = C\mathcal{E}_m \sin(\omega t)$.

c. $i_C(t) = \frac{dq(t)}{dt}$, so $i_C(t) = \omega C\mathcal{E}_m \cos(\omega t) = \frac{\mathcal{E}_m}{X_C} \cos(\omega t)$.

d. $i_G(t) = i_C(t) + i_R(t)$, so $i_G(t) = \mathcal{E}_m \left(\frac{1}{R} \sin(\omega t) + \frac{1}{X_C} \cos(\omega t) \right)$.

If you play around with trig formulas, you'll find that this is the same as

$$i_G(t) = \mathcal{E}_m \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}} \sin(\omega t + \phi) \quad \text{where} \quad \tan \phi = \frac{R}{X_C}.$$

Grading: 2 points for free; 2 points for each part. The “play around with trig formulas” mentioned in part (d.) is not required.