

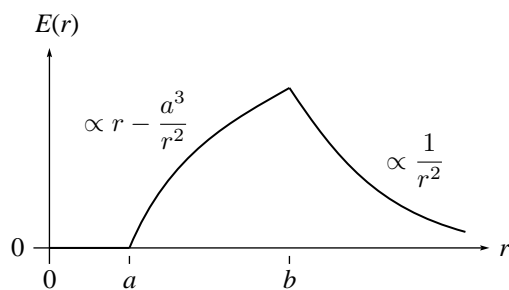
## The thick shell of charge

From the class discussion about “onions” of charge,

$$E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q_{[\text{inside } r]}}{r^2}.$$

So

$$E(r) = \begin{cases} 0 & r < a \\ \frac{1}{4\pi\epsilon_0} \frac{4}{3}\pi(r^3 - a^3) \frac{\rho}{r^2} & a < r < b \\ \frac{1}{4\pi\epsilon_0} \frac{4}{3}\pi(b^3 - a^3) \frac{\rho}{r^2} & b < r \end{cases}$$



The graphs of the left-most and right-most parts of the function are straightforward. For the middle portion ( $a < r < b$ ) note that the slope is proportional to

$$\text{slope} \propto 1 + 2\frac{a^3}{r^3}$$

so that (i) the slope is always positive — never zero or negative — and (ii) as  $r$  increases, the slope decreases.

*Grading:* 2 points for each part of answer equation ( $r < a$ ,  $a < r < b$ , and  $b < r$ ); 4 points for graph.