Experimental detection of magnetic monopoles

Griffiths, *Electrodynamics*, fourth edition, problem 7.39

Do you remember the Faraday law problems you did in Physics 111 involving changing magnetic flux?

$$\oint \vec{E} \cdot d\vec{\ell} = \mathcal{E} = -\frac{d\Phi_B}{dt}.$$

But also

$$\mathcal{E} = -L\frac{dI}{dt}$$

so

$$\frac{d\Phi_B}{dt} = L\frac{dI}{dt}.$$

Integrating both sides with respect to time,

$$\Delta\Phi_B = L\Delta I.$$

If the inital current is zero, then the final current will be $\Delta\Phi_B/L$. (Look up "snatch coil" or "flip coil".)

If magnetic monopoles exist, then Faraday's law changes to

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} - \mu_0 I_{m,enc} = -\frac{d\Phi_B}{dt} - \mu_0\frac{dq_m}{dt}.$$

where $I_{m,enc}$ is the enclosed current of magnetic charge.

If the loop is resistanceless, then $\vec{E} = 0$ inside the loop, so

$$\frac{d\Phi_B}{dt} = -\mu_0\frac{dq_m}{dt}.$$

Integrating both sides with respect to time

$$\Delta\Phi_B = -\mu_0 q_m,$$

where q_m is the amount of magnetic charge that has passed through the loop. If the initial magnetic flux is zero, then (remembering that $\Phi_B = LI$) the final current is

$$I = -\mu_0 q_m/L,$$

a quantity independent of the speed or direction of the magnetic charge!

Here's the physical picture: A moving magnetic monopole is ringed by $\vec{E}$ field, just as a moving electric charge is ringed by $\vec{B}$ field. When these rings approach a resistanceless loop of wire, they drive an electrical current within that wire. As this current springs into existence, it makes a changing magnetic field, and that changing magnetic field makes an electric field which counters the original $\vec{E}$ field of the moving magnetic monopole. Because the wire is resistanceless, the net effect must be to make the total $\vec{E}$ vanish within the wire. Once that current springs into existence, it remains forever (even when the initial magnetic monopole is far away) because the wire is resistanceless.

Note the cleverness of this approach: To find $I(t)$ you'd have to know the speed and direction of the magnetic monopole, the shape and area of the wire loop, and perhaps more…even thinking about this problem makes my head hurt. But all this is irrelevant to the final current.

My thanks to Ben Lemberger (took class in 2014) whose probing questions clarified this situation in my own mind.