

Maxwell stress tensor for light

Griffiths, *Electrodynamics*, fourth edition, problem 9.13

The wave in question is (see Griffiths equation 9.48)

$$\begin{aligned}\vec{E}(z, t) &= E_0 \cos(kz - \omega t + \delta) \hat{x} \equiv E_g(z, t) \hat{x} \\ \vec{B}(z, t) &= \frac{1}{c} E_0 \cos(kz - \omega t + \delta) \hat{y} = \frac{1}{c} E_g(z, t) \hat{y}\end{aligned}$$

(The subscript g stands for “given”.)

The components of the Maxwell stress tensor are given by Griffiths equation (8.17):

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right).$$

It is perfectly clear that all off-diagonal elements vanish. The three diagonal elements remaining are

$$\begin{aligned}T_{xx} &= \epsilon_0 \left(E_x E_x - \frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(-\frac{1}{2} B^2 \right) = \epsilon_0 \left(E_g^2(z, t) - \frac{1}{2} E_g^2(z, t) \right) + \frac{1}{\mu_0} \left(-\frac{1}{2} \left(\frac{1}{c} E_g(z, t) \right)^2 \right) = 0 \\ T_{yy} &= \epsilon_0 \left(-\frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_y B_y - \frac{1}{2} B^2 \right) = \epsilon_0 \left(-\frac{1}{2} E_g^2(z, t) \right) + \frac{1}{\mu_0} \left(\frac{1}{c^2} E_g^2(z, t) - \frac{1}{2} \left(\frac{1}{c} E_g(z, t) \right)^2 \right) = 0 \\ T_{zz} &= \epsilon_0 \left(-\frac{1}{2} E^2 \right) + \frac{1}{\mu_0} \left(-\frac{1}{2} B^2 \right) = -\epsilon_0 E_g^2(z, t)\end{aligned}$$

All the elements of the Maxwell stress tensor vanish, save for the element T_{zz} which equals $-u$ (the energy density).

This is a light wave moving in the z direction. The momentum of these fields is in the z direction, and the momentum is being transported in the z direction. If you’ve ever been swimming in the sea, you know that waves can knock you down (momentum transport). They knock you down in the direction they are traveling. This answer makes perfect sense.