# Water waves and quantal waves

**Griffiths *Electrodynamics*, fourth edition, problem 9.23**

**(a)** Deep water waves

$$\begin{aligned} v &= \alpha\sqrt{\lambda} \\ \lambda &= \frac{2\pi}{k} \\ v &= \frac{\omega}{k} = \alpha\sqrt{\frac{2\pi}{k}} \\ \omega &= \alpha\sqrt{2\pi k} \\ v_g &= \frac{d\omega}{dk} = \frac{1}{2}\alpha\sqrt{\frac{2\pi}{k}} = \frac{1}{2}v \end{aligned}$$

**(b)** Quantum mechanical waves
The form of the wave $Ae^{i(px-Et)/\hbar}$, compared to $Ae^{i(kx-\omega t)}$, tells us that

$$k = p/\hbar \quad \text{and} \quad \omega = E/\hbar,$$

so

$$E = \frac{p^2}{2m} \quad \text{means} \quad \hbar\omega = \frac{\hbar^2 k^2}{2m} \quad \text{or} \quad \omega = \frac{\hbar}{2m}k^2.$$

The wave velocity is

$$v = \frac{\omega}{k} = \frac{\hbar}{2m}k$$

while the group velocity is

$$v_g = \frac{d\omega}{dk} = \frac{\hbar}{m}k,$$

so

$$v_g = 2v.$$

The classical velocity is

$$\frac{p}{m} = \frac{\hbar k}{m},$$

so it corresponds to the group velocity.