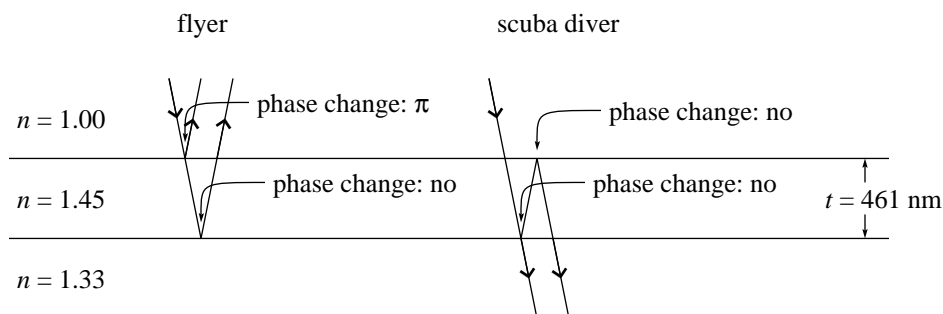


## Model Solutions to Second Sample Exam

### 1. Oil slick



	flyer	scuba diver	
constructive interference when	$2t = (m - \frac{1}{2})\lambda_{n=1.45}$	$2t = m\lambda_{n=1.45}$	for $m = 1, 2, 3, \dots$
	$\lambda_{\text{air}} = 1.45 \left( \frac{2t}{m - \frac{1}{2}} \right)$	$\lambda_{\text{air}} = 1.45 \left( \frac{2t}{m} \right)$	
$m = 1$ gives	$\lambda_{\text{air}} = 2670 \text{ nm}$ (infrared)	$\lambda_{\text{air}} = 1340 \text{ nm}$ (infrared)	
$m = 2$ gives	$\lambda_{\text{air}} = 891 \text{ nm}$ (infrared)	$\lambda_{\text{air}} = 668 \text{ nm}$ (red)	
$m = 3$ gives	$\lambda_{\text{air}} = 535 \text{ nm}$ (green)	$\lambda_{\text{air}} = 446 \text{ nm}$ (violet)	
$m = 4$ gives	$\lambda_{\text{air}} = 382 \text{ nm}$ (ultraviolet)	$\lambda_{\text{air}} = 334 \text{ nm}$ (ultraviolet)	

Thus the reflections of visible light are strongest for the flyer at 535 nm, and strongest for the diver at 668 nm and 446 nm.

### 2. Single slit diffraction curves

The intensity is

$$I(\theta) = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \text{where} \quad \alpha = \frac{\pi a}{\lambda} \sin \theta.$$

As  $\theta \rightarrow 0$ ,  $\alpha \rightarrow 0$ , and  $I(\theta)$  approaches its maximum, namely  $I_m$ . The half-maximum falls when  $I(\theta) = \frac{1}{2}I_m$ , that is when

$$\frac{1}{2} = \left( \frac{\sin \alpha}{\alpha} \right)^2,$$

that is when  $\sin \alpha = \alpha/\sqrt{2}$ .

### 3. Light bulb photons

*Strategy:* Find the amount of light energy pouring into your eye each second, then divide by the energy of each photon to find the number of photons. To implement this strategy, I'll need to estimate the energy of a typical photon, which comes from the wavelength of light typical from a light bulb.

*Implementation part A — light energy into eye:* The light bulb radiates light in all directions. A sphere 100 m away has an area of  $4\pi(100 \text{ m})^2$ . A pupil has area  $\pi(0.001 \text{ m})^2$ . So the fraction of the area taken up by a pupil is

$$\frac{\pi(0.001 \text{ m})^2}{4\pi(100 \text{ m})^2} = \frac{1}{4} \times 10^{-10}.$$

The 60-watt bulb sends out 60 joules each second, so in one second, one eye receives

$$(60 \text{ J}) \times \frac{1}{4} \times 10^{-10} = 1.5 \times 10^{-9} \text{ J}.$$

*Implementation part B — energy of a single photon:* I looked at Wikipedia and found a spectrum of a “typical incandescent lamp”: it had a peak near 650 nm. I'll use that as my “typical wavelength”. (You could have used any source, or even made a dead guess, but you have to *make your assumption clear* by stating what wavelength you're using.) The energy of one photon of this wavelength is

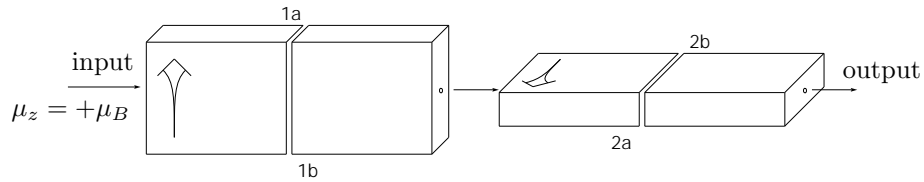
$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{650 \text{ nm}} = 1.9 \text{ eV} = 3.0 \times 10^{-19} \text{ J}.$$

*Implementation part C — number of photons:* The number of photons is thus about

$$\frac{1.5 \times 10^{-9} \text{ J}}{3.0 \times 10^{-19} \text{ J}} = 5 \times 10^9.$$

Clearly this is an estimate rather than a precise figure: Photons from the light bulb have a variety of energies, and your pupil is not exactly 1 mm in radius. So this number is not perfect, but it is infinitely better than the alternative “I have no idea”.

#### 4. Two more analyzer loops



If no paths are blocked, then all atoms take path 1a, no atoms take path 1b, and the atoms ambivate through both paths 2a and 2b.

If path 1a is blocked, then all the atoms are impaled on that blockage: 0% make it to output.

If path 1b is blocked, it has no effect because no atoms took that path to begin with: 100% make it to output.

If path 2a is blocked, half the atoms are impaled on that blockage: 50% make it to output.

If path 2b is blocked, half the atoms are impaled on that blockage: 50% make it to output.

If paths 1b and 2a are blocked, the blockage at 1b does nothing, and half the atoms are impaled on the blockage at 2a: 50% make it to output.

If paths 1a and 2b are blocked, the blockage at 1a impales all of them: 0% make it to output.

For full credit you must give an explanation, however brief, rather than simply state the numerical result.