Oberlin College Physics 212, Fall 2021

Model Solutions to Assignment 7

Characteristic quantities

I got numerical values and units from the site https://www.physics.nist.gov/cuu/Constants/index.html.

(a) First evaluate

$$\frac{e^2}{4\pi\epsilon_0} = \frac{(1.602 \times 10^{-19}\text{ C})^2}{4\pi(8.854 \times 10^{-12}\text{ F/m})} = 2.307 \times 10^{-28}\text{ N}\cdot\text{m}^2$$

then find the Bohr radius

$$\begin{aligned} a_0 &\equiv \frac{\hbar^2}{m_e(e^2/4\pi\epsilon_0)} = \frac{(1.055 \times 10^{-34}\text{ N}\cdot\text{m}\cdot\text{s})^2}{(9.109 \times 10^{-31}\text{ kg})(2.307 \times 10^{-28}\text{ N}\cdot\text{m}^2)} \\ &= 5.292 \times 10^{-11}\text{ m} = 0.052\,92\text{ nm} \end{aligned}$$

(Or you could go straight to the Bohr radius at the NIST site.) Blue light is about 450 nm, so the Bohr radius is about one ten-thousandths the wavelength of short-wavelength light.

⟦*Grading:* 1 point for numerical value; 1 point for correct number of significant figures (which will depend on number of significant figures in the data used); 2 points for comparison with blue light.⟧

(b) The period T of the innermost Bohr orbit obeys

$$\begin{aligned} \text{distance} &= \text{speed} \times \text{time} \\ 2\pi a_0 &= (p/m_e)T \\ T &= 2\pi a_0/(p/m_e) \end{aligned}$$

so the characteristic time is

$$\tau_0 = T/2\pi = a_0/(p/m_e).$$

But from equation (1.29) in the text,

$$p = \frac{m_e(e^2/4\pi\epsilon_0)}{\hbar}$$

so

$$p/m_e = \frac{(e^2/4\pi\epsilon_0)}{\hbar}.$$

Thus

$$\begin{aligned} \tau_0 &= \frac{a_0\hbar}{(e^2/4\pi\epsilon_0)} \quad (1) \\ &= \frac{(5.292 \times 10^{-11}\text{ m})(1.055 \times 10^{-34}\text{ N}\cdot\text{m}\cdot\text{s})}{2.307 \times 10^{-28}\text{ N}\cdot\text{m}^2} \\ &= 2.420 \times 10^{-17}\text{ s} = 0.024\,20\text{ fs} \quad (2) \end{aligned}$$

The period of blue light is 450 nm/c = 1.5 fs, about 60 times longer than this characteristic atomic time.

Atoms are a *lot* smaller and — what shall I say — have more *rumba* than light.

⟦*Grading:* 2 points for deriving expression (1); 1 point for numerical value; 1 point for correct number of significant figures (which will depend on number of significant figures in the data used); 2 points for comparison with blue light.⟧

Value of knowing characteristic lengths

The characteristic length for a hydrogen atom is the Bohr radius, 0.05 nm. Hydrogen sulfide is a bigger molecule, but it's not that much bigger. My guess for the diameter is 0.2 nm. A coronavirus, however, is a huge system with nucleic acids, a protein sheath, and spikes sticking out of the sheath. (Someone with a vivid imagination thought those spikes resembled a crown, hence the name "corona".) My guess is that it's a hundred times wider than a hydrogen sulfide molecule (and hence a million times the volume), so my guess is for a diameter of 20 nm.

Turning to the internet to check these guesses, the Wikipedia article on "Hydrogen sulfide" claims a size of about 0.1336 nm, a bit smaller than my guess. And the paper "Identification of Coronavirus Isolated from a Patient in Korea with COVID-19" at http://www.ncbi.nlm.nih.gov/pmc/articles/PMC7045880/ says that "Virus particle size ranged from 70–90 nm", a bit bigger than my guess.

By the way, a mask is not intended to filter out the coronavirus, but rather the water droplets upon which they reside.

⟦*Grading:* Hydrogen sulfide: 1 point for guess of length, 2 points for supporting argument (such as "not that much bigger"), 1 point for finding a value from the internet, 1 point for listing the source (such as "Wikipedia"). Coronavirus: 1 point for guess of length, 2 points for supporting argument (such as "a huge system"), 1 point for finding a value from the internet, 1 point for listing the source (such as "Identification of Coronavirus. . . "). Students will not make the same guesses that I did. . . any reasonable guess earns full credit.⟧

Questions (for chapter 2)

⟦*Grading:* 10 points for any decent attempt; 5 points for "I can't think of anything."; 0 points for no answer at all.⟧