**Oberlin College Physics 103, Fall 2023**

# Model Solutions to Assignment 5

Problems from *College Physics* by P.P. Urone and R. Hinrichs.

**Chapter 5, problem 4**

According to page 194, for wood on wood the friction coefficients are $\mu_s = 0.5$ and $\mu_k = 0.3$.

**Part (a).** For a mass of 120 kg, the weight is 1.18 kN, so the normal force is 1.18 kN, so the force needed to start the crate moving is $\mu_s N = 0.5$ kN.

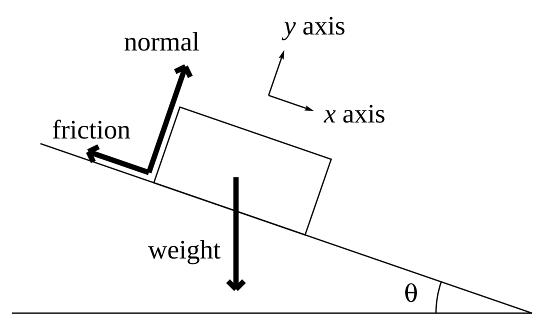
**Part (b).** That same force continues after the crate starts moving, and the friction becomes kinetic rather than static.

$$\begin{aligned}
\sum F &= ma \\
\mu_s N - \mu_k N &= ma \\
(\mu_s - \mu_k) mg &= ma \\
a &= (\mu_s - \mu_k) g = (0.2)(9.8 \text{ m/s}^2) = 2 \text{ m/s}^2
\end{aligned}$$

Note that this answer is independent of the crate's mass.

⟦*Grading:* Part (a): 2 points for general strategy, 2 points for number, 1 point for one significant figure. (500 N is technically wrong, but earns full credit.) Part (b): 2 points for general strategy, 2 points for number, 1 point for one significant figure.⟧

**Chapter 5, problem 9: Inclined plane with friction**




I will apply $\sum \vec{F} = m\vec{a}$, but instead of resolving the forces into horizontal and vertical, I will resolve them into "parallel to the plane" ($x$ direction) and "perpendicular to the plane" ($y$ direction).

$$\begin{aligned}
\text{parallel:} \qquad -f + mg\sin\theta &= ma \\
\text{perpendicular:} \qquad N - mg\cos\theta &= 0.
\end{aligned}$$

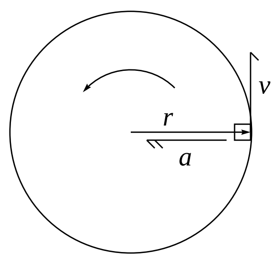
From the second equation, $N = mg\cos\theta$. Knowing that $f = \mu_k N = \mu_k mg\cos\theta$ allows us to write the first equation as

$$\begin{aligned}
-\mu_k mg\cos\theta + mg\sin\theta &= ma \\
g(\sin\theta - \mu_k\cos\theta) &= a.
\end{aligned}$$

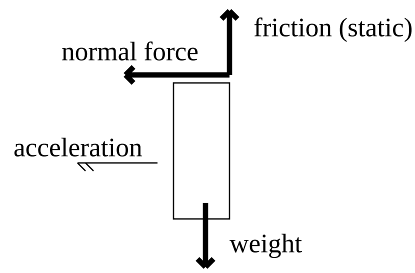
⟦*Grading:* 3 points for diagram, 4 points for resolving force components as show, 3 points for result. A student might resolve the forces horizontal and vertical, instead of parallel and perpendicular. If that student gets the correct result, he earns full credit, but has worked very hard to earn that full credit.⟧

**Chapter 6, problem 9: Construct your own problem: amusement park ride**

top view:




free body diagram for person, side view:




For the person, there is no acceleration (hopefully!), but the horizontal acceleration is $\omega^2 r$ (see textbook equation 6.17). The weight is $mg$, the normal force is called $N$. At minimum $\omega$ for dropping the floor, the friction is $f = \mu_s N$. Apply $\sum \vec{F} = m\vec{a}$ to the person:

$$\begin{aligned} \text{vertical:} \quad f - mg &= 0 \\ \text{horizontal:} \quad N &= m\omega^2 r. \end{aligned}$$

So at the minimum $\omega$ we have

$$\begin{aligned} \mu_s N &= mg \\ \mu_s (m\omega_{\text{min}}^2 r) &= mg \\ \omega_{\text{min}}^2 &= \frac{g}{\mu_s r} \\ \omega_{\text{min}} &= \sqrt{\frac{g}{\mu_s r}}. \end{aligned}$$

This equation has the correct dimensions. It makes sense that if $\mu_s$ increases (say by putting sandpaper on the walls), $\omega_{\text{min}}$ will decrease. But if $g$ were to increase (say by moving the ride to Jupiter) you'd have to increase $\omega_{\text{min}}$. Personally, I don't have any intuition about how $\omega_{\text{min}}$ should depend upon $r$.
⟦*Grading:* This is a "construct your own problem" so different students will go in different directions. Some will want to plug in numbers, some will want to discuss dependencies (as I did), some will want to draw more elaborate diagrams, some will just derive the equation. Any reasonable effort earns full credit of 10 points.⟧

**Chapter 6, problem 47: Dark matter**

This problem concerns the relationship between orbital period $T$, orbital radius $r$, and the mass of the body being orbited $M$. We didn't treat this matter in class, but page 253 of the text gives a good summary of section 6.6: these three quantities are related through

$$T^2 = \frac{4\pi^2}{GM} r^3,$$

where the gravitational constant is $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$. This is often called "Kepler's third law".

**Part (a).** *Question:* If $r = 6.0 \times 10^4$ light years and $M = 8.0 \times 10^{11}$ solar masses, what is the expected $T$?

*Solution:* Google will convert these into SI units for you: $r = 5.7 \times 10^{20}$ m; $M = 1.6 \times 10^{42}$ kg. Plug these into

$$T = \sqrt{\frac{4\pi^2}{GM} r^3} = 8.2 \times 10^{15}\ \mathrm{s}.$$

**Part (b).** *Question:* But in fact the period of a star at this distance is observed to be $6.0 \times 10^7$ years $= 1.9 \times 10^{15}$ s. What mass would cause such a period?

*Solution:*

$$M = \frac{4\pi^2 r^3}{GT^2} = 3.0 \times 10^{43}\ \mathrm{kg}.$$

*Discussion:* So it seems that the galaxy has a mass of $30 \times 10^{42}$ kg, but only $1.6 \times 10^{42}$ kg are luminous. The remainder (95%) is "dark matter".

⟦*Grading:* 4 points for quoting Kepler's third law; 3 points for successfully applying it in part (a); 3 points for successfully applying it in part (b).⟧