Oberlin College Physics 103, Fall 2023

Model Solutions to Assignment 7: Energy and Momentum

Problems from *College Physics* by P.P. Urone and R. Hinrichs.

Chapter 7, problem 21: Downhill ski race

Call the distance traveled along the slope d, the angle θ, and the height dropped $h = d\sin\theta$ To find final speed, use energy considerations:

$$\begin{aligned}\tfrac{1}{2}mv_i^2 + mgh &= \tfrac{1}{2}mv_f^2 \\ v_i^2 + 2gh &= v_f^2\end{aligned}$$

To find time required, use the equations (we haven't used these often: they're in the textbook at page 86)

$$\begin{aligned}x_f &= x_i + \bar{v}t \\ \bar{v} &= \tfrac{1}{2}(v_i + v_f)\end{aligned}$$

from which we conclude

$$\begin{aligned}d &= \tfrac{1}{2}(v_i + v_f)t \\ t &= \frac{2d}{v_i + v_f}\end{aligned}$$

Plugging in numbers, we find:
(a) When $v_i = 0$, $v_f = 26.2$ m/s, $t = 5.34$ s.
(b) When $v_i = 2.50$ m/s, $v_f = 26.3$ m/s, $t = 4.86$ s.
(c) So the final speed is nearly unchanged, but the time required decreases by almost 9%. This is because the boost in speed helps very much in the beginning of the race, although almost nothing at the end of the race.
⟦*Grading:* 2 points for v_f in (a), 2 points for t in (a), 2 points for v_f in (b), 2 points for t in (b), 2 points for any reasonable remark in (c). The problem statement used 30° when it should have used 30.0°, so students should get full credit if they report two significant figures.⟧

Chapter 7, problem 22: Subway train bumper

In the initial state there is kinetic energy due to train's motion but no potential energy due to spring compression. In the final state there is no kinetic energy but there is spring compression and hence spring potential energy. The energy is conserved:

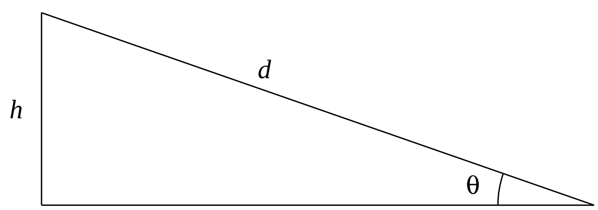
$$\begin{aligned}\mathrm{KE}_i &= \mathrm{PE}_f &(1)\\ \tfrac{1}{2}mv_i^2 &= \tfrac{1}{2}k\ell_f^2 &(2)\\ k &= m\left(\frac{v_i}{\ell_f}\right)^2 = 7.81\times 10^5\ \mathrm{N/m}. &(3)\end{aligned}$$

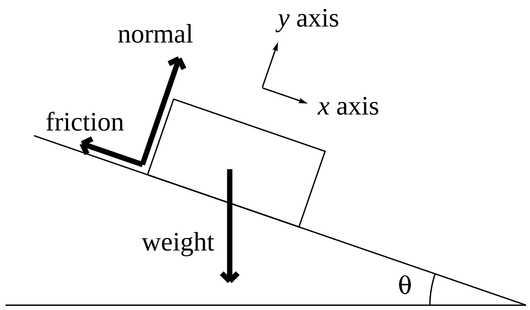
⟦*Grading:* 2 points for realizing that energy is conserved. 1 point for specific formulas for kinetic energy. 1 point for specific formula for spring potential energy. 2 points solving equation for k. 2 points for numerical value. 1 point for three significant figures. 1 point for units (the units kg/s^2 and J/m^2 are equally good).⟧

Hubba ledge

"A 78-kg skateboarder grinds down a hubba ledge 2.5 meters long and inclined 19 degrees below the horizontal. Half of her potential energy change is dissipated to sound or thermal energy through kinetic friction. What is the coefficient of kinetic friction between her skateboard and the ledge surface?"

At first glance it seems that this problem can't be solved, because initial and final velocities are not given.





Note from above right that $N = mg\cos\theta$.

$$\begin{aligned}
\text{work by friction} &= \tfrac{1}{2}(\text{potential energy change}) \\
-F_{\text{friction}}d &= \tfrac{1}{2}(\text{PE}_f - \text{PE}_i) \\
-\mu_k N d &= -\tfrac{1}{2}mgh \\
-\mu_k\, mg\cos\theta\, d &= -\tfrac{1}{2}mgd\sin\theta \\
\mu_k\cos\theta &= \tfrac{1}{2}\sin\theta \\
\mu_k &= \tfrac{1}{2}\tan\theta
\end{aligned}$$

Note the cancellation of m, g, d, and -1: the value $m = 78$ kg is not needed, the length of the hubba ledge $d = 2.5$ m is not needed, even the planet on which the experiment is performed (through g) is not needed! So, while my initial thought was that the problem statement didn't give all the information required, in fact it gave more than required! The answer is

$$\mu_k = \tfrac{1}{2}\tan(19^\circ) = 0.17.$$

⟦*Grading:* There are many ways to solve this problem, most of them less elegant than this, but any way that gives this result gets full credit. Partial credit should be assigned generously.⟧

Chapter 8, problem 12: Space debris

$$\begin{aligned} \text{impulse} &= \Delta\text{momentum} \\ Ft &= mv \\ F &= \frac{mv}{t} = \frac{(0.100 \times 10^{-6}\ \text{kg})(4.00 \times 10^{3}\ \text{m/s})}{6.00 \times 10^{-8}\ \text{s}} = 6.67 \times 10^{3}\ \text{N} \end{aligned}$$

⟦*Grading:* 4 points for reaching $F = mv/t$. 2 points for the number. 2 points for three significant figures. 2 points for units.⟧

Chapter 8, problem 44: Clown on skates

(a) By momentum conservation

$$\begin{aligned} 0 &= -m_C v_C + m_B v_B \\ m_B &= m_C \frac{v_C}{v_B} \\ &= (80.0\ \text{kg}) \frac{0.500\ \text{m/s}}{10.0\ \text{m/s}} \\ &= 4.00\ \text{kg}. \end{aligned}$$

(b) Kinetic energy at start is zero, at end is

$$\begin{aligned} \text{KE}_f &= \tfrac{1}{2} m_C v_C^2 + \tfrac{1}{2} m_B v_B^2 \\ &= \tfrac{1}{2}(80.0\ \text{kg})(0.500\ \text{m/s})^2 + \tfrac{1}{2}(4.00\ \text{kg})(10.0\ \text{m/s})^2 \\ &= 210\ \text{J}. \end{aligned}$$

(c) This kinetic energy comes from the chemical energy stored in glucose within the clown's body, which powers the muscle contractions and extensions needed to throw the barbell. If she keeps doing this, she will get tired and need to eat to replenish her glucose.
⟦*Grading:* 4 points for each of parts (a) and (b). 2 points for part (c). The answer to (c) does not need to be complete or eloquent, but there has to be something.⟧