Oberlin College Physics 103, Fall 2023

Model Solutions to Assignment 9: Rotation

Problems from *College Physics* by P.P. Urone and R. Hinrichs.

Chapter 10, problem 16: The torque of Zorch

Strategy: There are three stages to this problem:

> First, find the torque using $\tau = Fr$.
> Second, find angular acceleration using $\tau = I\alpha$
> Third, use the change in angular velocity through $\omega(t) = \omega_0 + \alpha_0 t$

I will use r_e for the earth's radius and m_e for its mass.

First stage: I'll call Zorch's force F_Z, so $\tau = F_Z r_e$. No need to put in numbers at this point.

Second stage: For the solid sphere earth (see table on page 397) $I = \frac{2}{5} m_e r_e^2$, so

$$\begin{aligned} \alpha &= \frac{\tau}{I} = \frac{F_Z r_e}{\frac{2}{5} m_e r_e^2} = \frac{5}{2} \frac{F_Z}{m_e r_e} \\ &= \frac{5}{2} \frac{4.00 \times 10^7 \text{ N}}{(5.97 \times 10^{24} \text{ kg})(6.38 \times 10^6 \text{ m})} = 2.62 \times 10^{-24} \text{ s}^{-2} \end{aligned}$$

Third stage: We are given rotational periods, not angular velocities, but it's easy to convert because $T = 2\pi/\omega$. The initial and final angular velocities are

$$\begin{aligned} \omega_i &= \frac{2\pi}{24.0 \text{ hr}} = \frac{2\pi}{8.64 \times 10^4 \text{ s}} = 7.27 \times 10^{-5} \text{ s}^{-1} \\ \omega_f &= \frac{2\pi}{28.0 \text{ hr}} = \frac{2\pi}{1.01 \times 10^5 \text{ s}} = 6.23 \times 10^{-5} \text{ s}^{-1} \end{aligned}$$

Use these in

$$\begin{aligned} \omega_f &= \omega_i - \alpha_0 t \\ t &= \frac{\omega_f - \omega_i}{-\alpha_0} \\ &= \frac{(6.23 - 7.27) \times 10^{-5} \text{ s}^{-1}}{-2.62 \times 10^{-24} \text{ s}^{-2}} \\ &= 3.97 \times 10^{18} \text{ s.} \end{aligned}$$

To get a better feel for this time you might convert it to 125 billion years. In contrast, the age of the universe (time to the big bang) is a mere 13.7 billion years.

⟦*Grading:* 2 points for starting out (for example, a figure, or a strategy), 2 points for each of the three stages, 1 point for unit "seconds" in final numerical answer, 1 point for three significant figures in final numerical answer. No need to convert to years, or to compare to the age of the universe.⟧

Chapter 10, problem 27: A bus with a flywheel

Using the table on page 397 we can find the moment of inertia of the flywheel (disk):

$$I = \tfrac{1}{2}M_{\text{flywheel}}R^2 = \tfrac{1}{2}(1500\text{ kg})(0.600\text{ m})^2 = 270\text{ kg}\cdot\text{m}^2.$$

(a) The translational energy required is

$$\tfrac{1}{2}M_{\text{bus}}v^2 = \tfrac{1}{2}(10,000\text{ kg})(20.0\text{ m/s})^2 = 2.00\times10^6\text{ J},$$

which demands

$$\begin{aligned} 0.900(\text{energy stored in flywheel}) &= 2.00\times10^6\text{ J} \\ \text{energy stored in flywheel} &= 2.22\times10^6\text{ J}. \end{aligned}$$

Since the energy stored is $\frac{1}{2}I\omega^2$, we have

$$\begin{aligned} \tfrac{1}{2}I\omega^2 &= 2.22\times10^6\text{ J} \\ \omega^2 &= \frac{2(2.22\times10^6\text{ J})}{270\text{ kg}\cdot\text{m}^2} = 1.64\times10^4\text{ s}^{-2} \\ \omega &= 128\text{ s}^{-1}. \end{aligned}$$

(This is 20.4 rotations per second: fast but not out of the question.)

(b) This part of the problem envisions the bus using the stored energy to go up a hill and still have speed 3.00 m/s at the top, that is

$$\begin{aligned} 0.900(\text{energy stored in flywheel}) - M_{\text{bus}}gh &= \tfrac{1}{2}M_{\text{bus}}(3.00\text{ m/s})^2 \\ h &= \left[0.900(\text{energy stored in flywheel})/M_{\text{bus}} - \tfrac{1}{2}(3.00\text{ m/s})^2\right]/g \\ &= 19.9\text{ m} \end{aligned}$$

⟦*Grading:* 1 point for finding I. 1 point for finding energy stored in flywheel. 2 points for finding number for ω, 1 point for units s^{-1}, 1 point for three significant figures. 2 points for finding number for h, 1 point for units m, 1 point for three significant figures.⟧

Chapter 10, problem 39: Grab onto a playground merry-go-round

We use conservation of angular momentum (kinetic energy is *not* conserved) between initial and final states:

$$I_i\omega_i = I_f\omega_f.$$

Because

$$f = \text{angular velocity in revolutions/second} = \frac{\omega}{2\pi}$$

there's no need to divide by 2π, just write

$$I_i f_i = I_f f_f.$$

Recalling the expressions for moment of inertia of a merry-go-round disk and for a single child placed at its rim, the above equation becomes

$$\left[\tfrac{1}{2}M_{\rm mgr}R^2\right] f_i = \left[\tfrac{1}{2}M_{\rm mgr}R^2 + M_{\rm child}R^2\right] f_f.$$

You see that the radius R cancels out right and left: it's irrelevant.

$$\begin{aligned}
\left[\tfrac{1}{2}M_{\rm mgr}\right] f_i &= \left[\tfrac{1}{2}M_{\rm mgr} + M_{\rm child}\right] f_f \\
f_f &= \left[\frac{M_{\rm mgr}}{M_{\rm mgr} + 2M_{\rm child}}\right] f_i \\
&= \left[\frac{120\ \text{kg}}{120\ \text{kg} + 2(22.0\ \text{kg})}\right] (0.500\ \text{rev/s}) \\
&= 0.366\ \text{rev/s}.
\end{aligned}$$

⟦*Grading:* 2 points for recognizing that angular momentum is conserved. 2 points for invoking the change in moment of inertia. 4 points for the number. 1 point for three significant figures. 1 point for units.⟧