Physics 103   Elementary Physics I

Model Solutions to Second Examination, 2023

1. *Red light*

A car of mass 1.53×10^3 kg is traveling at 8.94 m/s when the driver hits the break petal, supplying 3.82×10^3 N of breaking force. How far does the car travel before coming to rest?

Solution: This is an $\sum \vec{F} = m\vec{a}$ problem.

The only horizontal force is the breaking, so the horizontal acceleration is

$$\frac{F}{m} = \frac{-3.82\times10^3\ \mathrm{N}}{1.53\times10^3\ \mathrm{kg}} = -2.50\ \mathrm{m/s^2} \equiv a_0.$$

The relevant motion equation is the one connecting speed and displacement, without mentioning time:

$$\begin{aligned} v^2(x) &= v_0^2 + 2a_0(x - x_0) \\ 0 &= v_0^2 + 2a_0(\text{stopping distance}) \\ \text{stopping distance} &= \frac{v_0^2}{-2a_0} = \frac{(8.94\ \mathrm{m/s})^2}{5.00\ \mathrm{m/s^2}} = 16.0\ \mathrm{m} \end{aligned}$$

Alternatively, you can solve this problem using the work-kinetic energy theorem.

⟦*Grading:* 2 points for finding the $F = ma$ strategy; 2 points for finding a_0; 2 points for the kinematic equation for $v(x)$; 2 points for solving it for the stopping distance; 1 point for the number; 1 point for three significant figures.⟧

2. *Baseball*

A pitcher accelerates a 0.14 kg hardball from rest to 42 m/s in 0.062 s. How much work does the pitcher do on the ball?

Solution: Use the work-kinetic energy theorem. (The time required is irrelevant.)

$$\begin{aligned} \text{work} &= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \\ &= \tfrac{1}{2}(0.14\ \mathrm{kg})(42\ \mathrm{m/s})^2 - 0 \\ &= 0.12\ \mathrm{kJ} \end{aligned}$$

I will accept an answer of 120 J, although that last digit isn't really significant, but an answer of 123 J is wrong because the digit "3" is not significant.

⟦*Grading:* 2 points for finding the work-KE strategy; 3 points for the work-KE equation; 3 points for the number; 2 points for two significant figures.⟧

3. *Firecracker*

Two carts — a red cart with mass m_r and a blue cart with mass m_b — sit on the table with a small firecracker sandwiched between them. The firecracker explodes and the two carts fly apart from each other. Find an expression for the ratio of the kinetic energy of the red cart to the kinetic energy of the blue cart.

Solution: Momentum is conserved during the explosion. Total momentum is initially zero, so it has to be zero when they fly apart. They fly apart in opposite directions with speeds v_r and v_b such that

$$m_r v_r = m_b v_b.$$

Thus $v_r = (m_b/m_r)v_b$. The kinetic energy ratio is

$$\frac{\mathrm{KE}_r}{\mathrm{KE}_b} = \frac{\frac{1}{2}m_r v_r^2}{\frac{1}{2}m_b v_b^2} = \frac{m_r[(m_b/m_r)v_b]^2}{m_b v_b^2} = \frac{m_b}{m_r}.$$

⟦*Grading:* 2 points for recognizing momentum is conserved; 2 points for the momentum equation; 2 points for solving the momentum equation for v_r (or for v_b); 2 points for writing the KE equation; 2 points for solving it.⟧

4. *Rescue*

Tarzan (mass $m_T = 85.4$ kg) stands on a tree limb height $h = 12.4$ m above the Zambezi River and glances down to see a child in a canoe drifting slowly toward Victoria Falls. He grabs a convenient vine, swings down, and (while traveling horizontally) snatches the child out of the canoe. The Tarzan-child combination travels at speed $v_{as} = 13.1$ m/s immediately after the snatch. What is the mass of the child (m_C)?

Solution: This problem should remind you of the "projectile motion" lab.

Call Tarzan's speed immediately *before* the snatch v_{bs}.

Conservation of energy during the swing:

$$m_T g h = \tfrac{1}{2}m_T v_{bs}^2 \qquad \text{so} \qquad v_{bs}^2 = 2gh.$$

(You *could* plug in numbers to find $v_{bs} = 15.6$ m/s, but that's not required.) Conservation of momentum during the snatch:

$$m_T v_{bs} = (m_T + m_C)v_{as} \qquad \text{so} \qquad v_{bs} = (1 + m_C/m_T)v_{as}.$$

Eliminate v_{bs} between these two equations:

$$\begin{aligned} \sqrt{2gh} &= (1 + m_C/m_T)v_{as} \\ \frac{\sqrt{2gh}}{v_{as}} &= 1 + \frac{m_C}{m_T} \\ \left(\frac{\sqrt{2gh}}{v_{as}} - 1\right)m_T &= m_C \end{aligned}$$

Plugging in numbers, $m_C = 16.3$ kg.

⟦*Grading:* 1 point for recognizing energy conservation during the swing; 2 points for finding expression for v_{bs}. 1 point for recognizing momentum conservation during the snatch; 2 points for finding expression for v_{bs}. 1 point for equating the two expressions for v_{bs}, 1 point for solving for m_C, 1 point for plugging in numbers, 1 point for three significant figures.⟧