Model solutions for final exam; Elementary Physics I; Spring 2023

1. *Firecracker.* ⟦This problem was also on the second exam.⟧
Momentum is conserved during the explosion. Total momentum is initially zero, so it has to be zero when they fly apart. They fly apart in opposite directions with speeds v_r and v_b such that

$$m_r v_r = m_b v_b.$$

Thus $v_r = (m_b/m_r)v_b$. The kinetic energy ratio is

$$\frac{\mathrm{KE}_r}{\mathrm{KE}_b} = \frac{\frac{1}{2}m_r v_r^2}{\frac{1}{2}m_b v_b^2} = \frac{m_r[(m_b/m_r)v_b]^2}{m_b v_b^2} = \frac{m_b}{m_r}.$$

2. *What force?* The friction of the road acting on the wheels accelerates the car forward. The normal force exists but doesn't accelerate the car. The forces of engine on drive shaft and of exploding gas on piston exist and are necessary for the car's operation, but they don't accelerate the car: the force of the drive shaft on the engine is opposite to the force of the engine on the drive shaft. Only a force exerted *on* the car *by* something out of the car can make the car accelerate.

3. *Speeding up vs. slowing down.* For speeding up, use $v(t) = v_0 + at$ to find 3.0 m/s^2.
For slowing down, use $v^2(x) = v_0^2 + 2a(x - x_0)$ to find 11.9 m/s^2.

4. The change of momentum is zero, so the impulse delivered by the sandpaper must be the negative of the impulse delivered by me, namely (3.74 N)(5.67 s) = 21.2 N·s.

5. *Standing waves.* The half-wavelength is 2.14 m/5, so the full wavelength is 0.856 m. But $f\lambda = v$ so v is 16.6 m/s.

6. *Interference with sound.*
Complete constructive interference means distant loudspeaker is an integral number of wavelengths further than near loudspeaker. For the lowest frequency, i.e. longest wavelength, that's one wavelength:

$$\lambda = \sqrt{(3.38\mathrm{m})^2 + (2.69\ \mathrm{m})^2} - 2.69\ \mathrm{m} = 1.63\ \mathrm{m}. \quad \text{Then} \quad f = v/\lambda = 210\ \mathrm{Hz}.$$

7. *Hypothetical planet.* Use

$$F = m_p a = \frac{G m_s m_p}{r^2} \quad \text{so} \quad a = \frac{G m_s}{r^2} \quad \text{so} \quad r = \sqrt{\frac{G m_s}{a}}.$$

The mass of the planet is irrelevant. Plugging in numbers gives $r = 3.68 \times 10^9$ m.

8. *Drinks.*

$$\frac{V_{\text{below}}}{V_{\text{total}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{vodka}}} = 94.4\%.$$