

What is force? What does force do?

— Dan Styer, 19 September 2022

A force is a push or a pull.

Here in the physics department, a force does not necessarily come through a human or a living agent (although this *is* true of the “moral force” that they speak of in the religion department). In fact, the physics-style force does not necessarily involve contact! If I hold up a brick, then the Earth’s gravity pulls down on the brick, even though the Earth and the brick aren’t in contact. The evidence that the brick is pulled down is that if I drop the brick it falls.

But what if I don’t drop the brick — suppose that instead I set it on a pillow. Now the brick rests on the pillow. Even though gravity pulls it, the brick doesn’t fall. Why not? Because the pillow pushes up on the brick. You can see that the pillow pushes, because it compresses when I place the brick on it. (We say that the pillow exerts an upward force on the brick that precisely cancels the downward force of gravity. We say this even though the term “exerts” suggests that the pillow is huffing and puffing and getting tired like a weight lifter. The physics term “exerts a force” differs considerably from the more familiar athletics term of “exertion”.)

But what if I place the brick on a table top? In this case the table pushes up on the brick — but that seems wrong because the table, unlike the pillow, doesn’t seem to compress. In fact, the table *does* compress when a brick is placed on it, but the compression is very small. If we were to put one thousand bricks on a table we would notice a sag. Both the pillow and the table compress, but the table is much stiffer than the pillow so its compression is barely noticeable to the naked eye.

Building a force meter: an operational definition of force

This basic idea — force as a push or a pull — needs to be refined and made quantitative. One can refine the qualitative idea of “time” by building clocks of various accuracies; one can refine the qualitative idea of “distance” by building rulers, and Vernier calipers, and micrometers, and interferometers. In exactly the same way we will refine the qualitative idea of “force” by building force meters.

One way to measure forces is through rubber bands. I can loop a rubber band around my two thumbs, and place the thumbs ten inches apart. Now I can feel the rubber band pulling my two thumbs together. I could define this (operational definition) to be a force of one pound. To obtain a force of two pounds, I would stretch two identical rubber bands to a length of ten inches.

This is not just hypothetical. When my son wore dental braces, they were held in with “Tru-Force” brand orthodontic elastics (the technical term for rubber bands) that were calibrated to exert a force of 3.2 ounces when stretched to a length of 13/16 inch.

Now, rubber bands are not the most accurate force meters. For one thing, they tend to snap when they get old. For another, they’re great at measuring pulls but not so great at measuring pushes.¹ More accurate

¹A good way to measure pushes is by pushing through a pillow... we’ve already seen that a pillow compresses in order to exert a push.

force meters are made by springs: you can use delicate springs for measuring small forces, and stiff springs for measuring large forces. Your grocery store's spring scale is just a spring attached to a dial which measures the length of the spring.

At the moment, the most accurate force meter in the world is a device called the "proving ring". It is good for measuring both pushes and pulls. You can buy very accurate proving rings from the National Institute for Standards and Technology, just as you can buy very accurate kilogram masses from them. And for the same reason that NIST keeps its master kilogram in a vault where it can't rust or get dusty or be stolen, so it keeps its master proving ring in a similar vault.

[[An aside: In the physics laboratory the pound is a unit of force, even though in the grocery store the pound is a unit of quantity. This is because the force of gravity on a bunch of bananas is proportional to the quantity of bananas. Here on the surface of the Earth, that proportionality is used for commerce. If we did commerce on the surface of the Moon as well, we'd have to get used to the idea that a bunch of bananas that weighed three pounds on Earth would weigh half a pound on the Moon. The groceries we buy should really be doled out in kilograms rather than pounds, but I'm not going to get upset because I don't intend to take my bananas to the Moon. (The very most precise and pedantic people refer to this usage of "the pound" as the "the pound-force" and abbreviate it not as "lb" but as "lbf". This helps distinguish the pound as a unit of force from the pound as a unit of quantity from the pound as a British bank note.)

Here's an analogy: distances should be measured in miles or kilometers, but I commonly hear expressions like "Pittsburgh is three hours from Cleveland", meaning three hours of driving time. This statement is true when made in a bus station, but in an airport one frequently hears that "Pittsburgh is twenty minutes from Cleveland".]]

How does force affect motion?

Now that we know how to measure force, we can ask how force affects motion. Does force relate to velocity? To acceleration, the rate of change of velocity? To jerkiness, the rate of change of acceleration? To some combination such as $5v + 7a$?

Your first thought is that force causes velocity. After all, it's natural to say "If you push on something, it moves." But if you push a toy car on a table, it keeps on moving even after your hand stops pushing it. Furthermore, if the toy car is already moving, you have to push (backwards) on it to get it to stop. (Or else friction needs to push on it to get it to stop.)

In fact, experiment shows that force doesn't cause velocity, it causes acceleration.

Common: "If you push on something, it moves."

Correct: "If you push on something, its motion changes."

Galileo knew that this fact violated common sense, so he came up with not just one but two arguments to back it up. First, he said, suppose I have a very smooth track shaped like a V. A ball is released on the left arm of the V two feet above the trough. The ball descends to the trough, then keeps on going up the

other side until it's just a shade under two feet above the trough on the other side. Due to friction, it doesn't get all the way up to the two-foot mark, but the smoother you make the track the closer it will come.

Now suppose the right arm were not symmetric with the left, but that it went up with a shallower slope. Now the ball travels further along the right arm, but it still ends up just a shade below the two foot mark.

Finally, suppose the right arm doesn't slope upward at all, but just extends horizontally. The ball then would never be able to go up to the two-foot level where it started, and therefore it would go forever. If there is no force (including no frictional force) then there is no change in motion, but there certainly can be motion. Force is not needed to keep an object moving.

Here's Galileo's second argument. Suppose I toss a ball directly upwards. It flies straight up into the air, then comes down right into my waiting hand. There is no horizontal force, and no horizontal motion.

Now suppose I try the same experiment while inside a jetliner moving at 400 mph. (Galileo actually invoked a large sailing ship, but you're more likely to have traveled in a jetliner than in a large ship.) While I'm inside the cabin, I toss a ball directly upwards. It comes down right into my waiting hand. (It does not somehow get "stuck in the Earth's reference frame", in which case it would be moving backwards at 400 mph in the jetliner's frame!) In this case there is still no horizontal force, but there is horizontal motion at 400 mph.

How does force affect motion? — Quantitative

You can do experiments pulling objects using springs and rubber bands to measure the force exerted on the objects, and then measuring their acceleration. If you're doing these experiments on the Earth's surface, where gravity is present, it's easiest to exert the force horizontally and to measure the horizontal acceleration — this way you don't get confused by the gravitational force. And, like Galileo, do your best to make smooth surfaces so that extraneous friction forces don't confuse you. What do you find?

If a brick is pulled with one rubber band, stretched to five inches, and then pulled with two identical rubber bands, each stretched to five inches, then the second experiment results in exactly twice the acceleration. That's right: if we double the force we double the acceleration; if we triple the force we triple the acceleration. In general, the brick's acceleration is directly proportional to the force which, exerted on the brick, causes that acceleration:

$$a \propto F. \tag{1}$$

(It certainly makes sense that a should increase with F , but these experiments show more: that a increases linearly with F .)

So, what's the constant of proportionality? It's going to be different for different objects. Here's a neat argument about how the mass of the object will affect the acceleration.²

Suppose I pull a 3 kg brick with a rubber band stretched out to measure 2 pounds. The brick accelerates at, it turns out, 2.9 m/s².

²I thought that this argument was created by Galileo, but I've just checked a historical reference and I see that I'm wrong. I'm not sure who created it.

Now suppose I pull two of these bricks adjacent to each other, each with its own rubber band exerting 2 pounds. Of course each brick accelerates at 2.9 m/s^2 , so the two bricks stay right beside each other.

Finally, I do the same experiment but this time with a tiny amount of glue connecting the two bricks. Of course, I still have the same acceleration. But in this last case I have not two independent bricks, but one double brick of mass 6 kg pulled by two rubber bands for a total force of 4 pounds. Yet the acceleration must be 2.9 m/s^2 . In other words, if we double both the applied force and the mass, the acceleration is unchanged. Acceleration must therefore be proportional to the ratio

$$a \propto \frac{F}{m}. \quad (2)$$

So we've seen that the constant of proportionality in equation (1) depends on the mass of the object. What does the constant of proportionality in equation (2) depend on?

We can try various experiments: Does it depend on the object's color? No. On its height above sea level? No. On its chemical composition? (That is, does a kilogram of sulfur respond to force in the same way as a kilogram of uranium? A kilogram of hydrogen?) No. Despite the fact that sulfur, uranium, and hydrogen have very different hardnesses, strengths, densities, lusters, and so forth, they all respond to force in exactly the same way. Does it depend on the object's consciousness or vitality? No. A kilogram of stone behaves the same (in this respect) as a kilogram of living potted plant which behaves the same as a kilogram of sleeping squirrel which behaves the same as a kilogram of awakened squirrel. In fact, all experiments³ show that the constant of proportionality in equation (2) doesn't depend on anything else. When measured, this constant is found to be

$$a = k \frac{F}{m} \quad \text{where} \quad k = 4.448 \frac{\text{m kg}}{\text{s}^2 \text{ lb}}. \quad (3)$$

A second operational definition of force

In equation (3), we have three distinct ways to measure a , m , and F . This is the only way we could establish the result as a physical law and measure the constant k . However, once the law is established, we can use it to make a second operational definition of force.

The first operational definition embodies the qualitative idea of "force is a push or pull". Force is measured through a stretched spring: "A one pound force is the force exerted by a standard spring stretched to the one-pound mark." In this scheme, the constant k is measured.

The second operational definition embodies the qualitative idea of "force causes a change in motion". Force is measured through the amount of acceleration it causes: "A one pound force is the force that causes a one kilogram object to accelerate at 4.448 m/s^2 ." In this scheme the constant k is part of the definition of force, so this constant is defined rather than measured.

Well, if k is going to be defined, why not define it to have a convenient value? The convention is to define k to be 1, and in this case force is measured not in the unit of pounds, but in the unit of newtons, where a

³In spring 2007, Jens Gundlach and his colleagues at the University of Washington tested this principle at the precision of $5 \times 10^{-14} \text{ m/s}^2$. See S. Schlamminger, K.-Y. Choi, T.A. Wagner, J.H. Gundlach, and E.G. Adelberger, "Test of the Equivalence Principle Using a Rotating Torsion Balance", *Physical Review Letters*, **100** (2008) 041101.

newton is a $\text{kg}\cdot\text{m}/\text{s}^2$. The two units of force, newtons and pounds, are thus related through

$$1 \text{ lb} = 4.448 \text{ N}. \tag{4}$$

Let's review the strategy: (1) Make an independent operational definition of a new quantity. (2) Use that definition to discover a new law of nature. (3) Use that law to make a second operational definition of the quantity.

This three-part strategy is used all the time.⁴ You might think that at the end we could breath a sigh of relief and throw out all our standard springs, relieved that we no longer have to protect them from rust (and thievery). But in fact it's a good idea to keep the old standards around, just in case it happens that the law of nature you discovered is not quite right.

For example, the " $a = F/m$ " law turns out to be just a bit off — it turns out that while the proportionally constant in equation (2) doesn't depend on the color, height, vitality, or composition of the object under acceleration, it does depend on its velocity. We now believe that the correct law is

$$a = \left(\sqrt{1 - v^2/c^2} \right)^3 \frac{F}{m},$$

where c is the speed of light.⁵ As far as we know, there are no exceptions to this revised law (although its interpretation is delicate in quantum mechanical situations). Still, I wouldn't bet that we'll know of no exceptions 100 years from now. This shows that while $a = F/m$ can be used to provide a definition of force, that doesn't mean we've defined away a law of nature. Nature still has the upper hand.

⁴For example, there used to be an operational definition for heat resulting in the unit of "calorie". Using this definition, it was discovered that heat was a form of energy, and that one calorie was 4.184 joules. Now we measure heat in joules.

⁵Did I lie back above equation (3) when I said "all experiments show"? Yes, I lied. I couldn't think of any other way to put you into the right mood.