Oberlin College Physics 111, Spring 2024

Model Solutions to First Exam

1. The length of a diagonal is $\sqrt{2}a$. Half the length of a diagonal is $a/\sqrt{2}$.

a. Potential at center is

$$V_{\text{due to A}} + V_{\text{due to B}} + V_{\text{due to C}} + V_{\text{due to D}} = \frac{1}{4\pi\epsilon_0}\left[\frac{-2q}{a/\sqrt{2}} + \frac{+q}{a/\sqrt{2}} + \frac{+q}{a/\sqrt{2}} + \frac{-3q}{a/\sqrt{2}}\right] = \frac{1}{4\pi\epsilon_0}\frac{-3\sqrt{2}q}{a}.$$

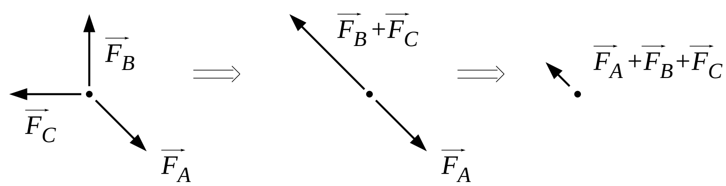
⟦*Grading:* Use of potential formula, 1 point. Correct answer, 2 points.⟧

b. From Coulomb's law,

$$|\vec{F}_A| = \frac{1}{4\pi\epsilon_0}\frac{(2q)(3q)}{(\sqrt{2}a)^2} = \frac{1}{4\pi\epsilon_0}\frac{3q^2}{a^2} \quad \text{while} \quad |\vec{F}_B| = |\vec{F}_C| = \frac{1}{4\pi\epsilon_0}\frac{3q^2}{a^2}$$

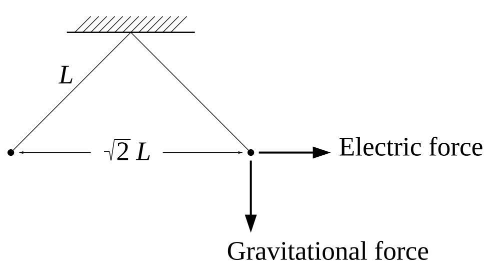
so

$$|\vec{F}_B + \vec{F}_C| = \frac{1}{4\pi\epsilon_0}\frac{3\sqrt{2}q^2}{a^2} \quad \text{and} \quad |\vec{F}_A + \vec{F}_B + \vec{F}_C| = \frac{1}{4\pi\epsilon_0}\frac{3(\sqrt{2}-1)q^2}{a^2}.$$



⟦*Grading:* Use of Coulomb's Law, 2 points. Correct magnitude, 3 points. Correct direction, 2 points.⟧

2. Electrostatics lab



It's clear from the figure that equlibrium comes when the electric force has the same magnitude as the gravitational force:

$$\frac{1}{4\pi\epsilon_0}\frac{q_L q_R}{(\sqrt{2}L)^2} = mg.$$

Solving for the product of charges gives

$$q_L q_R = \frac{2L^2 mg}{1/4\pi\epsilon_0}.$$

Plugging in numbers (remember: two significant digits!, convert to MKS!, state units of the final answer!) gives

$$q_L q_R = 5.3 \times 10^{-14}\ \mathrm{C}^2.$$

⟦*Grading:* Figure, 2 pts. Equation, 2 pts. Number, 2 pts. Two sig.figs., 2 pts. Units, 2 pts.⟧

3. Flux through the face of a cube

(1) Electric field is tangent to the top, right, and back faces, so for these faces $\Phi = 0$.
(2) The bottom, left, and front faces are arranged symmetrically relative to the charge.
(3) So each has the same flux: $\Phi_{\text{total}} = 3\Phi_{\text{front}}$.
(4) The charge inside the cube is $Q/8$.
(5) By Gauss's law, $\Phi_{\text{total}} = (Q/8)/\epsilon_0$ so $\Phi_{\text{front}} = Q/(24\epsilon_0)$.

⟦*Grading:* Each stage earns 2 points.⟧

4. Square of charge

$$E(z) = \frac{q}{4\pi\epsilon_0} \frac{z-a}{(z^2+a^2/4)(z^2+a^2/2)^{1/2}} \qquad (1)$$

A correct result would give $E(0) = 0$, but this one doesn't.

$$E(z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2+a^2/4)(z^2+a^2/2)^{1/2}} \qquad (2)$$

This one's correct.

$$E(z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2-a^2/4)(z^2+a^2/2)^{1/2}} \qquad (3)$$

Blows up at $z = a/2$. No way!

$$E(z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2+a^2/4)(z^2+a/2)^{1/2}} \qquad (4)$$

Dimensionally incorrect: expression in bottom right would give $[\text{length}]^2 + [\text{length}]$.

$$E(z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2+a^2/4)(2z^2+a^2/2)^{1/2}} \qquad (5)$$

A correct result must give E for a point charge when $a = 0$. This candidate doesn't.

⟦*Grading:* Each analysis earns 2 points.⟧