

## Model Solutions to Second Exam

**1. Teakettle.** The rate of water evaporation is proportional to the power dissipated at the resistive element, namely  $i^2R$ . If  $i$  increases by a factor of 1.21, then power dissipation increases by a factor of  $(1.21)^2 = 1.46$  (three significant figures), so the rate of water evaporation increases to (two significant figures)

$$1.46 \times (0.41 \text{ cup/min}) = 0.60 \text{ cup/min.}$$

[[*Grading:* 2 points for “rate proportional to power”; 2 points for  $i^2R$ ; 2 points for number; 2 points for two significant figures; 2 points for “cup/min” (either explicit or in text).]]

**2. Network circuit.** Call the current up through the left battery  $i_L$  (this is our desired quantity), the current up through the right battery  $i_R$ . The loop rule applied to the left loop gives

$$13 \text{ V} - i_L(15 \Omega) - (i_R + i_L)(22 \Omega) = 0$$

while the loop rule applied to the right loop gives

$$25 \text{ V} - i_R(11 \Omega) - (i_R + i_L)(22 \Omega) = 0.$$

So we need to solve simultaneously

$$\begin{aligned} 13 \text{ V} - i_L(37 \Omega) - i_R(22 \Omega) &= 0 \\ 25 \text{ V} - i_L(22 \Omega) - i_R(33 \Omega) &= 0 \end{aligned}$$

To eliminate  $i_R$  multiply the top equation by 3, and the bottom equation by  $-2$ , then sum:

$$(3 \times 13 - 2 \times 25) \text{ V} - i_L(3 \times 37 - 2 \times 22) \Omega = 0$$

giving

$$i_L = \frac{(3 \times 13 - 2 \times 25)}{(3 \times 37 - 2 \times 22)} \text{ A} = \frac{-11}{67} \text{ A} = -0.16 \text{ A.}$$

Note the negative sign. The 25 V battery is so hefty that it actually forces current *down* through the 13 V battery.

[[*Grading:* 2 points for starting off; 2 points for left loop equation; 2 points for right loop equation; 4 points for solution.]]

**3. Force on table wire.** Because  $\vec{F} = i\vec{L} \times \vec{B}$ , where  $\vec{L}$  is horizontal, the horizontal component of  $\vec{B}$ , parallel to  $\vec{L}$ , does not contribute to the force. The magnitude is  $F = iLB_v = 0.474$  mN, and the direction (through right-hand rule) is horizontal, toward magnetic west (i.e. toward  $7.60^\circ$  south of west).

[[*Grading:* 2 points for correct equation; 4 points for correct magnitude; 4 points for correct direction.]]

**4. Switched-on circuit.** Immediately after the switch is closed, no current flows through the inductor.

- (a) Current through the  $10 \Omega$  resistor is  $(30 \text{ V})/(30 \Omega) = 1 \text{ A}$ .
- (b) So the voltage drop across the  $10 \Omega$  resistor is  $10 \text{ V}$ , whence the emf of the inductor is  $10 \text{ V}$ , whence  $di/dt = \mathcal{E}/L = (10 \text{ V})/(0.2 \text{ mH}) = 50 \text{ kA/s}$ .

After a long time has passed, changes have stopped happening so the inductor has nothing to fight. It acts like a simple piece of wire.

- (c) Current through the  $10 \Omega$  resistor is zero — it shunts through the simple piece of wire instead.
- (d)  $di/dt = 0$  — everything has settled down and is not changing with time.

[[*Grading:* 3 points each for (a) and (b); 2 points each for (c) and (d).]]