Private Reading, Fall 2024

Assignment 13

Reading: Griffiths chapter 12 on "Electrodynamics and Relativity".

Also *Notes on Electrodynamics* chapter 8, "Relativistic Electrodynamics" and chapter 9, "Atmospheric Optics".

Problems: Think about the following four problems, but none of them are required.

1. Units in electromagnetism. Available on course website.

2. Field energy and momentum. We know the relativistic frame transformation laws for time and space, for energy and momentum, and for electric and magnetic fields. We also know expressions for electromagnetic energy and momentum in terms of electric and magnetic fields. Show that electromagnetic energy and momentum *do not* transform as energy and momentum ought to transform. (Produce expressions for the total electromagnetic energy and momentum within a region in the "barred" frame, in terms of those quantities, plus the electric and magnetic fields, in the "unbarred" frame.)

If you can figure out a way to resolve this conundrum, that would be great too. Otherwise, I'll do my best to resolve it when we meet the stress-energy tensor.

3. New derivation of relativistic momentum expression. Follow the collision approach of "Notes on Relativistic Dynamics", chapter 2, but instead of equation (2.15) write

$$p = mf(v)v$$

where $f(0) = 1$. Determine which function(s) $f(v)$ will result in momentum conservation in all frames.

I have not yet solved this problem so I'll be lenient in grading any good faith effort. Student Stella Ocker, class of 2018, recommends the article by P.C. Peters, "An alternate derivation of relativistic momentum," *American Journal of Physics* **54**, 804–808 (September 1986). My teacher N.D. Mermin also liked to approach special relativity through this approach of "arbitrary functions that are then constrained", but I can't find any of his papers that address this specific point.

4. Front-back symmetry of the field from a uniformly moving charge. At equation (7.10) in the notes, I remarked on the front-back reflection symmetry of the electric field produced by a uniformly moving charge. Our mathematics shows that symmetry is definitely there, but it's hard to understand: it's as if a ship moving in water produced not only a wake behind the stern but also a wake in front of the bow! Hengrui Zhu, class of 2021, suggested this approach: Lorentz transform into a reference frame where the charge is stationary, find the electric field in that frame where there is a clear spherical symmetry, then Lorentz transform back. Implement this suggestion and see whether it gives a clear picture of the origin of the front-back reflection symmetry.