Electric induction

Griffiths, *Electrodynamics*, fourth edition, problem 7.8

a. For a long straight wire

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(directed out of the page)}$$

so for this square

$$\Phi_B = \int \vec{B} \cdot \hat{n}\, da = \int_s^{a+s} \frac{\mu_0 i}{2\pi r}\, a\, dr = \frac{\mu_0 i a}{2\pi} \left[\ln r\right]_s^{a+s} = \frac{\mu_0 i a}{2\pi} \ln \frac{a+s}{s} = \frac{\mu_0 i a}{2\pi} \ln(1 + a/s).$$

b. The emf is given by the flux rule, Griffiths equation (7.13),

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 i a}{2\pi} \left[\frac{d}{ds} \ln(1 + a/s)\right] \frac{ds}{dt} = -\frac{\mu_0 i a}{2\pi} \left[\frac{-a/s^2(t)}{1 + a/s(t)}\right] v = \frac{\mu_0 i v}{2\pi} \frac{(a/s(t))^2}{1 + (a/s(t))}.$$

By Lenz's law (discussion at bottom of page 308 and top of page 309), current flows counterclockwise.

c. No change in flux, so no emf, so no current.