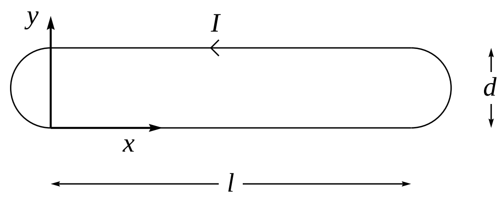
Inductance of a hairpin loop

Griffiths, *Electrodynamics*, fourth edition, problem 7.25



If the current is I, then inside the loop

$$\vec{B}(x,y) = \left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(d-y)}\right)\hat{k}.$$

Clearly, the $\vec{B}$ due to the top wire doesn't equal the $\vec{B}$ due to the bottom wire, yet the flux due to the top wire *does* equal the flux due to the bottom wire. The total flux is

$$\Phi_B = 2\frac{\mu_0 I}{2\pi}\ell \int_0^d \frac{1}{y}\,dy = \frac{\mu_0 I \ell}{\pi}[\ln y]_0^d = \frac{\mu_0 I \ell}{\pi}[\ln(d/0)] = \infty.$$

Oops. As Griffiths suggests, we must take account of the finite wire width ϵ. To do this exactly, we would have to know the distribution of current density throughout the wire, and find the field due to this distribution both within and outside the wire. This would be very hard. But a simple approximate way of tackling the problem is to integrate, not from 0 to d, but from ϵ to $d - \epsilon \approx d$. The result is

$$\Phi_B = \frac{\mu_0 I \ell}{\pi}[\ln(d/\epsilon)]$$

whence

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 \ell}{\pi}\ln(d/\epsilon).$$