A Sample Physics Problem

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What are effective techniques for solving physics problems? How can solving problems help you understand physics? What does your teacher expect to find in your solutions? This document answers these questions through an example. It shows in action the principles described in the document "Solving Problems in Physics".

The problem

Here is our sample physics problem: (This problem is based on problem **2**–37 in David Halliday, Robert Resnick, and Jearl Walker, *Fundamentals of Physics*, sixth edition, 2001, page 29, which is identical to problem **2**–35 in David Halliday, Robert Resnick, Jearl Walker, and Karen Cummings, *Fundamentals of Physics, Alternate Edition*, 2001, page Pr-5.)

> **2**–35. *Red light.* A car moves at constant speed when a traffic light ahead turns red. After a brief reaction time, the driver steps on the break petal and then the car slows with constant deceleration to a stop. The car requires 56.7 meters to stop from a speed of 80.5 km/hour, and 24.4 meters to stop from a speed of 48.3 km/hour. What is the reaction time of the driver and the rate of deceleration due to breaking? Discuss your numerical answer and the equations that lead up to it.

Finding a solution

What kind of problem is this? It's a hybrid of two problems: first (during the "reaction phase") it's a uniform speed problem, second (during the "deceleration phase") it's a uniform deceleration problem. Let's refresh our understanding of these two types of motion through the chart below:

Relation:	uniform speed (v_0)	uniform deceleration (d)
v to t	$v = v_0$	$v = v_0 - dt$
x to t	$x = x_0 + v_0 t$	$x = x_0 + v_0 t - \frac{1}{2} d t^2$
v to x	$v = v_0$	$v^2 = v_0^2 - 2d(x - x_0)$

(The deceleration d is the negative of the acceleration a.)

Where are we? Where do we want to go? This table outlines the situation, and at the same time defines the variables we'll use:

	reaction phase	deceleration phase
time required:	t_R	t_D
distance traveled:	x_R	x_D

We know $(x_R + x_D)$ and v_0 — we want to find t_R and d. It's clear that no one of the six equations above will do the job for us.

Strategy, first try. However, we can put together equations. For example, the two "relate x to t" equations can be applied to this situation as

$$x_R = v_0 t_R \qquad x_D = v_0 t_D - \tfrac{1}{2} d t_D^2.$$

And these two can be summed to produce

$$x_R + x_D = v_0(t_R + t_D) - \tfrac{1}{2} d t_D^2.$$

This equation looks useful: it relates things we know ($x_R + x_D$, v_0) to things we want to find (t_R, d), with only one extraneous quantity, namely t_D. (By "extraneous" I mean a quantity that we don't know and that we don't want to find.) You might think that we could write this equation once for the first case (56.7 meters from 80.5 km/hour) and once for the second case (24.4 meters from 48.3 km/hour), then eliminate t_D from the two resulting equations. This strategy fails because the deceleration time t_D will be different in the two different cases.

Strategy, second try. Since we don't know and don't care about the two different values of t_D for the two cases, let's not use the relation between x and t for the deceleration phase. Instead, we'll try the relation between v and x for this phase, namely

$$v^2 = v_0^2 - 2d(x - x_0)$$

which in our situation becomes

$$0 = v_0^2 - 2dx_D \quad \text{or} \quad x_D = \frac{v_0^2}{2d}.$$

Combining this with our previous equation $x_R = v_0 t_R$ for the reaction phase gives

$$x_R + x_D = v_0 t_R + \frac{v_0^2}{2d}.$$

This looks like what we need! It has no extraneous quantities. Plugging in known numbers for the two cases will produce two equations, and solving the two equations simultaneously will find the two unknown quantities. Let's rush out and do it. . .

Check. No. Slow down. Before going through the labor of plugging in numbers, let's check this equation for reasonableness. The dimensions are correct. The result will be positive. If the reaction time t_R increases, then the stopping distance $x_R + x_D$ increases. If the initial speed v_0 increases, then the stopping distance increases. If the deceleration d increases, then the stopping distance decreases. All of this makes sense. There's a brief moment of panic when we ask "What if $d = 0$? Then the resulting stopping distance is infinite!" But that's okay: if there's no deceleration, then the car never stops, so the stopping distance *should* be infinite. The result passes all our checks for reasonableness.

Plug in. Now we've arrived at a good place to put in numbers. Those numbers are:

	$x_R + x_D$	v_0
first case:	56.7 meters	80.5 km/hour = 22.4 m/sec
second case:	24.4 meters	48.3 km/hour = 13.4 m/sec

(Note the conversion to SI units.) So using

$$x_R + x_D = v_0 t_R + \frac{v_0^2}{2d}.$$

for the two cases we have (in meters and seconds):

$$\begin{aligned} 56.7 &= 22.4\, t_R + \frac{(22.4)^2}{2d} \\ 24.4 &= 13.4\, t_R + \frac{(13.4)^2}{2d}. \end{aligned}$$

Our plan is to eliminate the deceleration d first. To do this, divide the first equation by $(22.4)^2$ and the second by $(13.4)^2$, giving

$$\begin{aligned} 0.113 &= 0.0446\, t_R + \frac{1}{2d} \\ 0.136 &= 0.0746\, t_R + \frac{1}{2d}. \end{aligned}$$

Subtraction eliminates the deceleration:

$$0.023 = 0.0300\, t_R$$

or

$$t_R = 0.767 \text{ sec.}$$

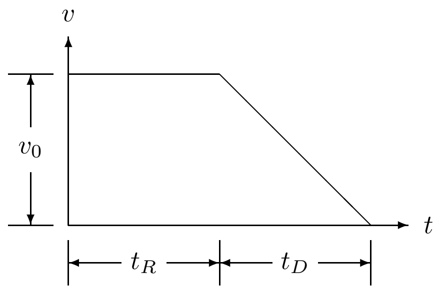
And plugging this into either of the two equations involving deceleration gives

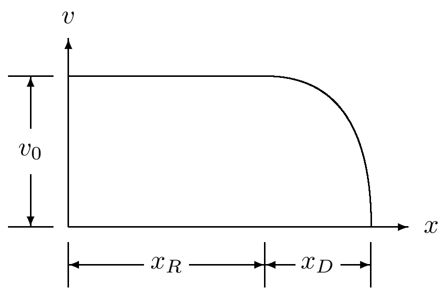
$$d = 6.34 \text{ m/sec}^2.$$

Both of these numbers seem reasonable to me: the reaction time is just under one second, and the deceleration is somewhat less than the acceleration of gravity $g = 9.8$ m/sec^2.

⟦True confession: The first time I did this problem I miscopied the stopping distance of 56.7 meters — I wrote 76.7 meters instead. I knew I had made an error when I got a negative reaction time.⟧

Graphs. Although the problem does not require this, it is informative to sketch graphs of v versus t and of v versus x.





Why do these graphs show t_R equal to t_D but x_R greater than x_D? Because the car is going faster during the reaction phase than during the deceleration phase, so if the times were equal then the reaction phase distance would be greater.

Writing up a solution

Your write-up doesn't need to describe all the blind alleys you went down in arriving at a solution. And it doesn't need to be flowing literate prose. But it does need to (1) show your reasoning, not just give the answer; (2) describe your thoughts in words and in figures as well as in equations; and (3) outline your checking. (When the problem statement says to "discuss", it usually means to outline your checks for reasonableness.)

The next page gives a full-credit answer to this problem. Things to note: (1) Define your symbols. (2) Choose mnemonic names for your symbols. (3) Don't present every arithmetic step (you're not in training to become a pocket calculator!), but do present every logical step. (4) Give numerical results with units and with the proper number of significant digits. (5) Include your name!

Problem **2***–35: Red light.* *Dan Styer*

Initial speed v_0 maintained during reaction phase, followed by constant deceleration d.

	reaction phase	deceleration phase
time required:	t_R	t_D
distance traveled:	x_R	x_D

Known: $(x_R + x_D)$ and v_0
Desired: t_R and d.

Use $x = v_0 t$ during reaction phase: $x_R = v_0 t_R$

Use $v^2 = v_0^2 - 2d(x - x_0)$ during deceleration phase: $0 = v_0^2 - 2dx_D$ or $x_D = \frac{v_0^2}{2d}$.

Add these two:

$$x_R + x_D = v_0 t_R + \frac{v_0^2}{2d}.$$

Checks:

Dimensions okay.
Stopping distance $x_R + x_D$ is positive.
Stopping distance increases with reaction time.
Stopping distance increases with initial speed.
Stopping distance decreases with deceleration.
Correct result $x_R + x_D = \infty$ for special case $d = 0$.

Plug in numbers:

	$x_R + x_D$	v_0
first case:	56.7 meters	80.5 km/hour = 22.4 m/sec
second case:	24.4 meters	48.3 km/hour = 13.4 m/sec

So for the two cases:

$$\begin{aligned} 56.7 \text{ m} &= 22.4 \text{ m/sec } t_R + \frac{(22.4 \text{ m/sec})^2}{2d} \\ 24.4 \text{ m} &= 13.4 \text{ m/sec } t_R + \frac{(13.4 \text{ m/sec})^2}{2d}. \end{aligned}$$

Solving these two equations in two unknowns gives

$$t_R = 0.767 \text{ sec}, \qquad d = 6.34 \text{ m/sec}^2.$$

Check:
Numbers are reasonable: the reaction time just under one second, deceleration somewhat less than g.